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Original Article

Exploring Students' Proficiency in Mathematical Symbolization

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ABSTRACT

This study examined the students' proficiency in using mathematical symbols. The need to carry out the study was prompted by the Kenya National Examinations Council (KNEC) report that revealed dismal performance in mathematics. Therefore, the study objectives explored how symbols affect learning of mathematics, students' perception of the role of symbols in mathematics learning and students' use of mathematical symbols. The basis of the study relied on a conceptual framework of epistemological approach to notations and supportive and problematic conceptions as a lens that helped in dissecting the kind of symbol sense that exist amongst students. The study targeted mathematics teachers and form four students and was therefore conducted in a public secondary school in Rarieda Sub-County, Siaya County, Kenya. A qualitative approach with a case study research design was employed with sampling techniques such as convenience, purposive, and stratified sampling used to locate the research site and recruit participants. Data collection instruments included interview guides and document analysis protocol. Thematic analysis was used. The findings of the study showed that symbols influenced the learning of mathematics in terms of prior knowledge and symbol meanings at hand, thereby posing challenges in the learning of mathematics. Also, the findings revealed that students had a perception of the role of mathematical symbols in giving easy time in understanding concepts due to their precise and succinct nature, conserved time and assisted in the solution of mathematics problems and that use of symbols is profound in the linkage of concepts across topics, multiple representations and problem-solving. It may be recommended that prominence ought to be put on various ways of symbol representation to enable comprehension of symbols and meanings; better instructional techniques ought to be used to reduce the symbol cognitive load on students.

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INTRODUCTION

Mathematics is a major subject in the curriculum that is compulsory in primary and secondary schools. Mathematics is expressed in symbols (Kharde, 2016) and makes use of symbols in communication (Bardini & Pierce, 2015). Secondary mathematics consists of several symbols such as equal sign, plus sign, minus sign, division, multiplication, letters, and brackets that express and represent concepts (De Cruz & De Smedt, 2013). Learners must develop a conceptual understanding of symbols in linking respective symbols to their concepts. Rubenstein & Thompson (2001) posit that terminology and vocabulary help reveal mathematical ideas. In the Kenyan context, research on mathematical language, terminology, and vocabulary has been done with limited research, specifically on mathematical symbols. Mbugua's (2012) research findings revealed that conceptual understanding and achievement are determined by mathematical language. Also, the student's difficulties in mathematics were witnessed in areas of mathematical terms and related concepts, and possible remedies were suggested (Mulwa, 2014). However, the Kenya National Examination Council (KNEC) report for the last five years still indicates dismal performance in mathematics despite the research and suggestions made on the impact of mathematical language on achievement.

Table 1: KCSE Data Report 2015-2019.

Year	2015	2016	2017	2018	2019
Mean score	26.9	20.8	25.5	26.4	27.5

Source: KNEC report 2020

From the foregoing, it means that the challenge has not been addressed. It is therefore imperative to conduct research specifically on proficiency in using mathematical symbols.

Statement of the Problem

There has been poor performance in mathematics in the history of the Kenya National Examination Council (KNEC), which has continued for several years. Secondary mathematics comprises algebra and geometry, which have a myriad of symbols forming the basis of communication in mathematics. These symbols are representative of concepts that may not be conceived easily without the symbols (De Cruz & De Smedt, 2013). It is expected that students' interaction with symbols is smooth and with less friction so as to yield better achievement. However, that is not the case in mathematics classes, as inefficiency and inability to use symbols are witnessed (Jiew & Chin, 2020). Learners do not associate symbols with their concepts and meanings (Nogueira de Lima & Tall, 2008) and have challenges in using mathematics terms and concepts (Mulwa, 2015).

Therefore, the study sought to investigate the extent of proficiency by students in the use of symbols in mathematics for conceptual understanding in Rarieda Sub-County, Siaya County, Kenya. The research findings may provide a platform for teachers and teacher educators to come up with executable mathematical instruction that can improve students' conceptual understanding of mathematical symbols.

Research Questions

The study sought to answer the following questions: The main question: How do students use symbols to conceptually understand mathematics? The specific questions were: How do symbols affect learning of mathematics? How do students perceive the role of symbols in

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mathematics learning? How do students use mathematical symbols?

LITERATURE REVIEW

Mathematical Symbolization and Proficiency

A significant number of literature reviewed informed about the symbolization, proficiency in mathematical symbolization, and the conceptual underpinning for the study. Mathematics as a discipline has been making use of symbols from time immemorial, both in the realist and non-realist views of mathematics. Symbolism has endeavoured to propel concepts and conceptual understanding. According to Cobb (2002), mathematical symbols are representatives of mathematical ideas, objects, concepts, or processes. The representative is related through the process of symbolization (Godino & Batanero, 2003). Representation being constitutive of symbolism has had great significance in both the teaching and learning of mathematics in areas such as numbers, operations, signs and algebraic expressions. Symbols manifest in attributes such as materiality, syntax, and meaning, where materiality refers to the appearance of the symbol, syntax is the unity between the symbol and the rules, also known as the symbol system and meaning being the sense relayed by the symbol (Serfati, 2005). Moschkovich (2006) also posited that the language of mathematics stems from the symbols that comprise syntax, the symbols organization, and the language of instruction.

Mathematical symbolization is therefore a process that gives meaning and relationships between mathematics ideas, objects, concepts, or processes and those that have been conventionally agreed upon. This may be depicted in contexts of problem-solving and multiple representations revealing levels of proficiency in individuals (Kaput & Shaffer, 2002) though there are expected challenges. These challenges may include "Polysemy symbols" –symbols with many meanings (Mamolo, 2010), different symbols representing the same concepts, the 'procept' nature of symbols-symbols representing both concepts and process (Gray & Tall, 1994), and contextual meaning (Mamolo, 2010).

Mathematical proficiency in symbolization borrows a lot from some of the Kilpatrick et al. (2001) strands of proficiency specific to conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. For the purpose of this study, the strands have been collapsed into one major proficiency that is referred to as representational fluency. Fonger (2019) defined representational fluency as a continuum of meaningful understanding of mathematics that Skemp (1976) referred to as relational understanding. This involves the linking of representation from symbolic to other representations (Pierce & Stacey, 2004). An individual has to be equipped both in symbol sense and capacity so as to be able to create, interpret, and make meaning of symbolic representation in solving problems (Arcavi, 1994; Sullivan, 2013). This may be exemplified in reading, writing, and verbalizing symbols.

Conceptual Framework

This study used an epistemological approach to mathematical notations (Serfati, 2005) and supportive and problematic conception (Jiew & Chin, 2020) as lenses to give an insight into the conceptual grounding for the study.

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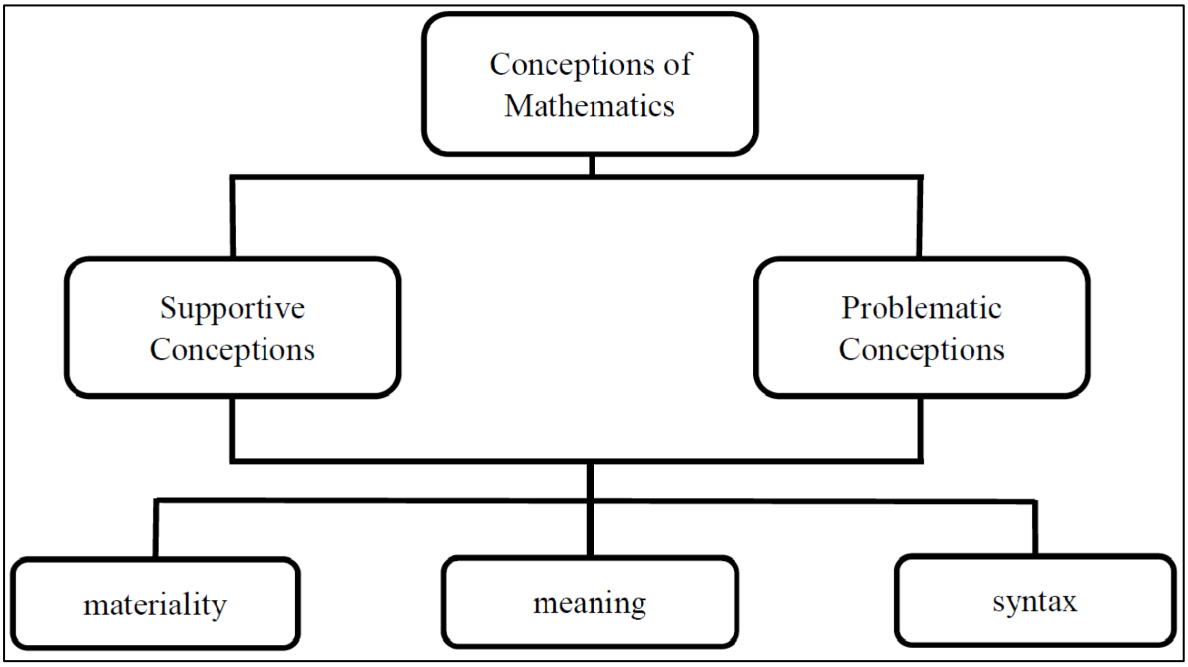
Figure 1: Conceptual Framework.



Source: Adopted from Chin & Pierce (2019)

As illustrated in *Figure 1*, conceptions involve internal representation and associations evoked by the concept that may be either supportive or problematic (Tall, 2013). When a conception operates smoothly in both old and new contexts, then it is supportive; otherwise, it is a problematic conception when it hinders progress. The conception is key to the mathematical symbolization process that may manifest according to the epistemological approach to mathematical notations. This considers that the symbols have attributes of symbol load that are dichotomized into symbol density and familiarity. This framework is anchored on the symbol familiarity that consists of materiality, syntax, and meaning. In the conception of symbols, an individual must be able to recognize these attributes and the contexts under consideration (Bardini & Pierce, 2015). The conception process is affected by prior experiences that are referred to as "met-before" (Nogueira de Lima & Tall, 2008). Therefore, the conception may be supportive met-before in case it influences knowledge development positively and problematic met-before if it influences knowledge development negatively. The illustration depicts that conception is a dependent variable that may feature as problematic or supportive depending on context and symbol. On the other hand, symbol familiarity, syntax, and meaning are independent variables as these attributes are conventional and have standard specific conceptions expected.

METHODOLOGY

Research Approach and Design

The study employed a qualitative approach to explore the student's proficiency in the use of mathematical symbols. The approach was chosen as a result of the conveniences that are associated with it, especially on interaction with the respondents during the research (Akinyode & Khan, 2018) and its ample opportunity for in-depth exploration of a problem under study (Abdullahis & Senekal, 2012). The case study research design used focused on a single unit of a school (Lodico et al., 2006). It is believed that the chosen research design provided room for data triangulation hence boosting the trustworthiness (Crowe et al., 2011).

Sample and Sampling Procedure

The participants of this study were sampled from two schools in the Rarieda sub-county and that was done on convenience sampling for ease of access (Robinson, 2014), providing an accessible population. The Rarieda sub-county is vast and therefore reaching the participants had to be pegged on availability and accessibility. The

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participants were selected purposively as mathematics teachers with at least eight years of experience in teaching mathematics and students in form four classes due to their experience also with high school mathematics. This was arrived at from Scott & Morrison's (2006) revelation that it is at the discretion of the researcher to pick whom to engage in the sample. The study was confined to 2 teachers and eight students were used as participants in the study.

Data Collection Methods, Instruments and Procedure

Two methods of data collection were used. They included semi-structured interviews and document analysis. This agrees with Creswell (2009) who posits that triangulation of data makes the study thorough, rich, and in-depth with less biases. Further, the creation of convergence and corroboration was enhanced (Bowen, 2009). The researcher conducted a semi-structured interview with each of the 10 participants guided by the interview guide consisting of a list of questions. The interviews were audio recorded and helped in obtaining verbatim responses for transcription during data analysis (Busetto et al., 2020).

Data from students' exercise books were also examined to reveal the use of symbols. The analysis was guided by the document analysis protocol that had constructs that revolved around the topic under study. This was attributed to the unobtrusive and unreactive nature of documents (Bowen, 2009)

Data Analysis

The data analysis followed an iterative process. The collected data was transcribed verbatim, where the audio recorded data was reproduced into written form (McGrath et al., 2019). The produced transcripts were put together with captured field notes and the document analysis protocol. Descriptive coding that involved open and axial coding was used where development of initial codes was done through open coding. Further, axial coding was then used to refine the codes and generate themes (Given, 2008; Walliman, 2010). Codes and themes were repeatedly examined so that the research questions of the study were answered.

RESULTS

The findings revealed that symbols affect the learning of mathematics; students had a perception of the role of symbols in the learning of mathematics and the use of symbols in areas of linkage across topics, multiple representations, and problem-solving.

Influence of Symbols on Mathematics Learning

Prior Knowledge of Symbols

The findings established that prior knowledge by learners about symbols influenced their mathematics learning since perspectives of conception emerged as supportive and some problematic conceptions. On knowledge of symbols, a student had to say that some of the examples of symbols were the sigma (**Σ**) and equal (=) sign. The same student maintained that the equal sign had the following implication:

> *Maybe when comparing equal variables. When you have done mathematics, when finalizing your answer, you use an equal sign... that an answer is derived.* [Personal communication, FM1, September 8 2020].

The illustrative data extracts above show students had knowledge about symbols that they used in their day-to-day mathematics, which consisted of letters, numerals, and other operator signs such as addition, subtraction, multiplication, and division. Other reactions that came about were:

> *We can just say addition and subtraction, it is just easy. For multiplication and division, I am just good at them.* [Personal communication, Student FM2]

The students coped differently with symbols as some of the challenges depicted ranged from easy to difficult. It may be argued that mathematics learning would be progressive or impeded by prior knowledge that students had about the respective symbols. The symbols that had a connection with others in higher classes would be used with a bit of prior knowledge. However, problematic conception arises when mathematical symbols are met for the first time and have no linkage to other symbols in secondary mathematics.

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Meaning of Symbols

The findings established that multiple meanings of mathematical symbols hamper students' learning of mathematics owing to prevailing context and procepts. For example, the equal (=) sign symbol that was mentioned, others had the belief that it implied that an answer was reached. In cases where the symbol system was involved with multiple symbols, such as $(a)\ 2+3=\ and\ (b)\ 2+3=1+4$, the student had to interpret the meaning of the equal sign (=) in this manner:

> *It means this* $2+3$ *equals or is the same as the answer you are going to get, while* $2+3=1+4$ *is that the sum of 2 plus 3 and 1 plus 4 are the same. It is going to give you one answer.* [Personal communication, MM3]

The same symbol system had the plus (+) symbol that was commonly interpreted to have a singular meaning as that of the addition of two digits.

A plus (+) sign in 2+3 implies adding the two [Personal communication, Student FM2]

> *I can connect some of the symbols to their meanings; like + in the 3+4 implies addition; +5 implies you are adding +5, adding 5 times* [Personal communication, Student FM4]

The symbol meaning is out rightly an issue in the learning of mathematics since most of the students could tell the single meaning of symbols, not knowing that other possible meanings existed. The association of symbols to their meaning in given contexts is challenging to students since some gave meanings that are applicable to other different symbols hence bringing confusion. This may be said to have caused an inability to link the representation of various symbols to the concept and process. Also, mathematical symbols may possess different meanings in considerable contexts like +5.

Student MM2 affirmed during the interview that the symbols are of utility for conceptual understanding and may not be used anyhow but depending on task demands as illustrated:

> *I just apply those symbols where necessary. According to the question. Some do not require the use of symbols* [Personal communication, Student MM2]

The problem provided dictates what is to be done together with the symbols involved. There is a thin line between the mathematics symbols and concepts and this was confirmed by the teacher participants when asked about how the cultivation of symbols meanings is done during the instructional process as follows:

> *Mathematics and symbols are inseparable, any moment I give instruction or interact with the learners or teach mathematics to the learners, symbols and mathematics is part of it* [Personal communication, Teacher T090].

The mastery of a symbol directly affects the learning of mathematics. Keen learners with good mastery of symbol meanings find it easier to do mathematics. However, symbols' meaning is the cause for jittery reactions during the students' interaction with mathematics. This is revealed in their answers that for conceptual understanding to take place, the connection of symbol meanings and concepts has to be done hence simultaneity in mathematical symbolization, concept, and processes.

Students' Perception of Role of Symbols in Mathematics Learning

The Usefulness of Symbols in Solving Mathematics Problems

Mathematics derives its power from the symbols used. This has impacted a great deal on the conceptual understanding of mathematics in general. Mathematics concepts are expressed with clarity and facilitate correspondence in thinking. Attributable coding, construction, and communication of mathematics language happen through symbols. This may be affirmed from the responses given by the students during the interview about the usefulness of symbols that:

> *I think they give me an easy time. Maybe let us say you are given a sum to calculate, there is no need to even write an equal sign in full, you use a sign to conserve time* [Personal communication, student MM3]

Another student had to say that:

> *They help us solve mathematics…. They are easy if you know the content.* [Personal communication, student FM3]

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These data extracts reveal a lot about learners' knowledge of the role of symbols in mathematics as illustrating the structure of mathematics, manipulating routines, and reflection. However, little reservations were about the mastery of the mathematics content that is believed to be simultaneous with symbols. Further, the student's responses from learners were reinforced by the teacher's report that symbols have made the mastery of mathematics concepts easy with bare brains, unlike if symbols were not used. During the interview, teachers had to say that:

> *The symbol as used in mathematics is meant to reduce the cognitive load on the learner's mind and the symbols tend to be brief and can also be easily compared to word statements. So, reducing the cognitive load in the mind of the learner is a major role of symbols in mathematics* [Personal communication, TeacherT090]

They say mathematics made easier; they ease the understanding of the mathematics topics. [Personal communication, teacher T010]

Attitude and Belief

The findings showed that students' perception towards the learning of mathematical symbols is that continuous effort is mandatory for effective learning. This was brought from the responses given out by the students during the interview. They listed a number of strategies that ought to be applied, such as a steady revision process for mastery of concepts and processes. For example, student FM 1 had to say that:

> *I revise mathematics regularly so that I do not forget. Some symbols I normally cram* [Personal communication Student FM1]

The student went further to say that some symbols do not require conceptual understanding and that instrumental understanding, like cramming was favourable in order to achieve good grades in mathematics.

Another student, MM3 maintained that the learning of mathematics symbols involved a routine that one must set in order to master symbol concepts and processes represented. These routine processes were augmented by other psychomotor demonstrations of some symbols that are illustrated from the reporting during the interview with the student:

> *In any case, maybe I am doing mathematics or maybe am active, I make sure that every day I learn one symbol… some symbols I just practice them manually without doing them like greater than (>)* [Personal communication Student MM3]

The hate and belief that mathematical symbols do not help students in the learning of mathematics were also registered among a few individuals, which showed that impedance in perception was generated through it. It seems that attempts at conceptual understanding may not make headway in mathematics classes since the existing situation has little attachment to symbols for learning mathematics. The student MM1 had a considerable number of thoughts about symbols though later gave the utility attached and the need to go look back on symbols as a remedy, especially on reading symbols.

> *The only problem is that from Form 1 up to Form 4 we have not been reading those particular things, I hate them; they cannot even help us by the way… the solution is that I should go back and check them* [Personal communication Student MM1]

This hate may not have come so singly but contributions derived from less effort that instructors are putting into ways of representing symbols for conceptual understanding may be the cause. Otherwise, the usefulness of symbols in the learning of mathematics ought to have come out due to representation simultaneity between symbols, concepts, and processes.

Students' Use of Symbols

Students' use of symbols in mathematics manifested in three different areas: linkage of symbols across topics, multiple representations, and problem-solving**.**

Linkage of Symbols Across Topics

The ability to link symbol use across topics in mathematics is a measure of proficiency in symbol use and further conceptual understanding. The findings established that learners could use one symbol in a topic and link it further to other topics where its use is appropriate. This was revealed from the transcribed data that showed the reactions as follows:

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> *In F1, we have algebra that can be used in the topic of integration in F4 or just calculus as a whole, so we can borrow. Like identities ($x^2 + 5$) you can use those particular identities to differentiate* [Personal communication Student MM1]

In F1 linear inequalities using symbols such as greater than, less than; the symbols apply in F4 topic under linear programming [Personal communication Student MM2]

It came out vividly that learners could give highlights of a topic with specific symbols that were appropriate in succeeding and preceding the topic. The topics such as linear inequalities and linear programming in forms two and four, respectively, are dependent on each other such that the knowledge of linear inequalities was highly applicable in linear programming (KIE, 2002; KLB, 2008).

Multiple Representation

The data reveals that learners used symbols to link representation from one form to another and further switch representations. The manifested representations were in reading, writing, and spoken. During the interview, students were tasked to say the kind of symbol representations that they were conversant or proficient with, and student MM1 reported that:

> *No, I have not been reading symbols…I think it is not a bit technical, but I can read it…writing math symbols is easy* [Personal communication, student MM1]

The same student had difficulty speaking out about the symbol system: $\int(2x^2 + x + 2)dx$ Which was spoken as integrate $2x^2 + x + 2$ in terms of dx and later said that verbalizing symbols is not easy. The difficulties witnessed in student MM1 above were again realized in student MM2 when tasked to do the reading of $\frac{dy}{dx}$, writing and verbalizing of $\int(2x^2 + x + 2)dx$ and the report was that:

> *Yes, I can read. That is $\frac{dy}{dx}$. That is a gradient function or derivative… I can write symbols… $\int(2x^2 + x + 2)dx$ this means you are integrating y-equation with reference to dx…. verbalizing is somehow difficult* [Personal communication, student MM2]

The students showed good proficiency in their reading and writing of symbols though with challenges in speaking out the symbols. This was noted from the kind of errors and misconceptions that students had. The use of symbols in multiple representations was marred with mixed abilities as some could do specific representations in specific domains as compared to others. The data collected also depicted that switching of representation could be done effectively, especially from symbolic representation to pictorial representations. For example, in the representation of the equation of a straight line to other representations like tabular and graphical, students elaborately gave a sequence of steps to express the equation of a straight line graphically as follows:

> *You make a table of x and y, then you put x and put in the equation to get the value of y; then you equate where there is x you put zero and y will be 1; then you choose another value of x may be one, you substitute in the equation. Then draw the plane, mark the points; join to get the line* [Personal communication, Student MM3]

> *First of all, you must sketch something like a table…then you plot x and y after that. Then you can be able to have x values, after which you are going to replace in that particular equation so as to get the y-axis. So, after getting the x and y axis now you can plot them in a graph so that you can sketch a graph. Change of representation from one form to another is tasking* [Personal communication, student MM1]

The two data extracts gave a glimpse of the use of symbols in representation among the students revealing representational fluency. The students had clear procedures of what happens in every step of involvement of symbol use. Further, these outcomes are supported by the working on exercise books that showed the graphical representation for a waveform from the numerical representation in *Figure 2*.

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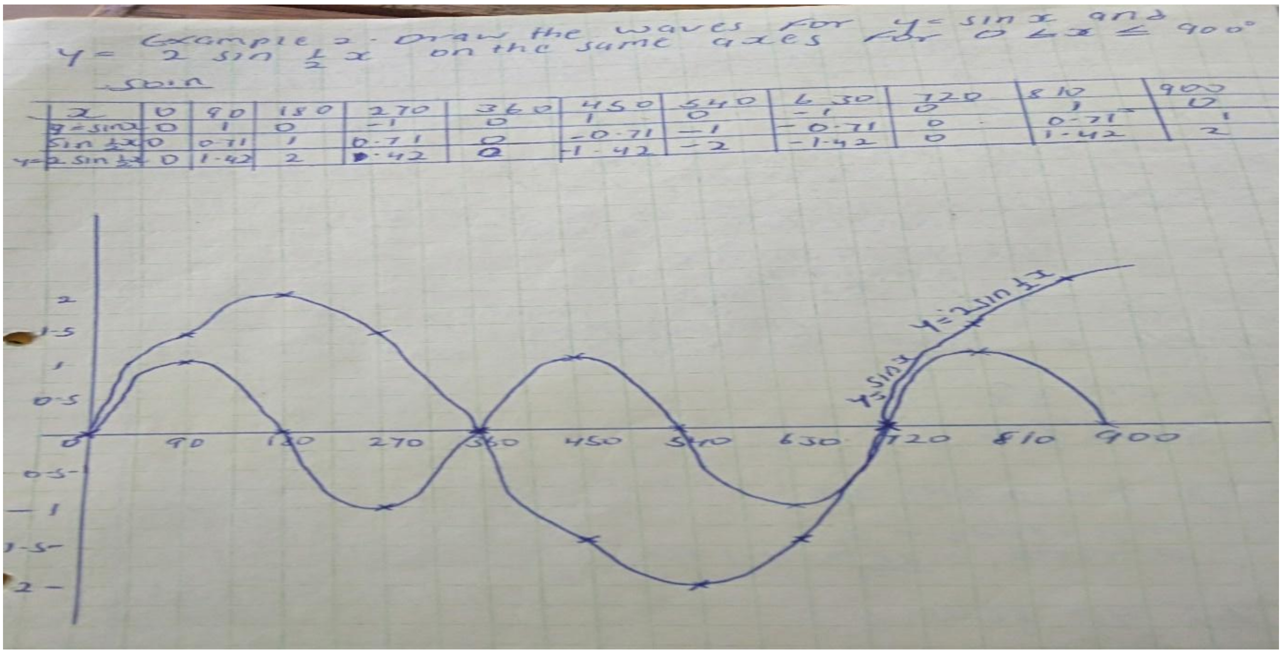
Figure 2: Work of MM2 on Graphical Representation from the mathematics exercise books



Problem Solving

The findings showed that students used symbols during problem-solving of class assignments and examinations. The use of symbols on such occasions influenced their representation of problem situations, procedures, goals, activities, and the organization of results. These cases were confirmed by interviewed students who reported that their use of symbols in areas of problem-solving was adequate though the time frame was not regular as it is dependent on the occasion only:

> *Yes. I can do it when the teacher leaves us assignments, during examinations or during my personal revision* [Personal communication, Student MM2]

However, others contradicted their colleagues that the symbol used is not a routine in problem-solving, which is not the case. The established findings have it that there is the awareness that symbols play a big role in influencing how a mathematical problem is approached, the laid activities, and the results structure, as shown by the transcribed data from interviews of students.

> *What I can say if you have been given an exercise. To tackle you may be in a bad or good position. For example,* $\frac{dy}{dx} = 3x + 2$ *, that one you integrate* [Personal communication, student MM4]

Student MM4 admits that a symbol in a specific problem may pose a difficulty or a breakthrough in the solving of a problem. This, therefore, reaffirms the power of symbols that mathematics acquires and that problem solving is a precursor to symbol conceptual understanding.

The symbol density may also have an impact on solving a problem since all the symbols have to be subjected to interpretation and their relationship with the symbol system. For instance, in the case of an interview with student MM3 to explain how symbols influence the solution of equation $3x + 8 = 15$, the relayed explanation was that:

> *They change the expression; the symbols in the equation are =, +; can make it easy to cross take eight the other side and the sign changes* [Personal communication, teacher MM3]

The teacher T090 also had to give a case of students in class on how they would perceive the symbol influence in problem-solving and the kind of goals, activities and organization of work that may be involved during the solution of the problem. The following data came out:

> *The x as a symbol there would appear unique to the learner because the learner looks at it and sees that the rest are figures, but x is not known and would automatically imagine that my duty is to find out what this x is. So already,*

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> *it probes the learner* [Personal communication, teacher T090]

DISCUSSION

The findings established that symbols influenced the learning of mathematics attributable to prior knowledge of symbols and symbol meanings. Prior knowledge about symbols influenced the way symbols were used in mathematics learning by students. Some of the symbols that are interacted with at lower levels, such as equal and plus signs, were mostly referred to, and their uses were based on how they were associated with them. This conception of symbols from prior knowledge revealed a lot of misconceptions yielded from problematic conception (Tall, 2013). However, the list of symbols mentioned was within an epistemological approach to mathematical notations as consisting of letters, figures, and compound templates (Serfati, 2005). The fondly applied symbols were those whose background information had been derived owing to supportive conception. However, the others whose interactions were only within the level of the student coping up hit a snug as they had no prior experience with them. Also, for those who had knowledge about misconstruing took place since the underlying implication of symbols was never revealed. The challenge may have resulted from the fact mathematical language relies on the use of symbols that had no association with the background knowledge of the learners.

The meaning of symbols was found to as hampering the learning of mathematics. Studies have shown that symbols may have multiple meanings depending on contexts in what is referred to as procept (Gray & Tall, 1994). The dual nature of mathematics symbols is revealed in both process and concept, but the student participants had mastered and hung on single meanings, posing a great challenge to their learning of mathematics. Such interpretation of singular meanings ought to have not been the case with symbols such as the equal (=) sign and plus (+) sign that learners have met on several occasions. This contrasted with the findings from a research study by Mamolo (2010) that a symbol meaning revealed in some contexts may not be in agreement with other usual contexts. In the Kenyan context, a research study done by Mulwa (2015) has it that there were difficulties in the use of mathematical terms and concepts. These concepts are simultaneous, with symbols posing equal challenges in the use of symbols. Therefore, the challenges of symbol meanings contribute greatly to the way mathematics is done. Symbols are known for attributes such as symbol density and familiarity (Bardini & Pierce, 2015). The textbooks contain a considerable number of symbols; hence symbol load also other attributes like the “symbol physique” and the syntax in the symbol system creates alternative meanings that students may not be able to reveal. Many have resorted to using symbols with little or no conceptual understanding, otherwise known as “symbol pushing”. This ought to have not been the case with a common symbol like the equal (=) sign, which has representations of operation, relation, and specification meanings. The learners maintained that the equal sign had the operator symbol meaning only since that is what all along has been their interaction. This concept of “met before” hampered the thinking such that students may not go outside what they have known and consider other meanings that prevail (Jiew & Chin, 2020).

The symbols play an important role in the learning of mathematics which manifests in two folds: the usefulness of symbols and the aspect of attitude and belief. This prevailed as a result of the student’s perception of the effect that symbols have on the conceptual understanding of mathematics. The findings indicated that mathematical symbols gave easy time in understanding concepts due to their precise and succinct nature, conserved time and assisted in the solution of mathematics problems. The findings were in tandem with other research findings by Chirume (2012) that symbols had a pivotal role in communication, saving time and brief organization. This further amplified the usefulness of mathematical symbols in the learning of mathematics.

Perceived attitudes and beliefs towards mathematical symbols were revealed. The findings indicated that students had the notion that continuous effort was mandatory for effective learning of mathematics symbols and mathematics in general. Therefore, some set routine has to be followed in order to steadily apply effort. Other contrasting opinions came about that in cases where conceptual understanding is not possible, then preference may be taken for rote learning that only develops instrumental understanding (Skemp, 1976). These findings are reinforced by other research study

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findings that showed that belief and attitude held by students is a function of efficacy in mathematics (Kilpatrick et al., 2001). Mweni's (2015) findings also had to say that attitude affects performance in mathematics, thereby agreeing with the findings of this research study about attitude. Though some farfetched perception that symbols do not help in the learning of mathematics owing to the hate of symbols also came about.

Findings revealed the potency of the linkage of symbols across various topics in the secondary mathematics syllabus. Students were able to link symbols from one related topic to another, such as linear inequalities and linear programming topics. This showed internalized symbol sense which Arcavi (1994) said determined the choice for use or abandoning of a symbol for a specific context.

Multiple representations were common among the students though they varied among individuals. Students used symbols to link representation from one form to another that testified in speech, reading, and writing. The featured forms of representation were symbolic, numerical, tabular, and graphical representations which had equations of a straight line to be represented in tables and graphs. Representation ensured that students expressed abstract mathematical ideas in concrete forms (PrayiTno et al., 2020). This showed the testimony of representational fluency in terms of speech, reading, and writing among learners. The only domain of testimony that had weakness was the speech that a considerable number of students failed in, but this did not interfere with representation from numerical, tabular, or graphical and vice versa. The study by Ulusoy & Argun (2019) on the examination of high school students revealed the reason for inadequate verbal representation that is due to poor reading.

Problem-solving is a mode of learning mathematics symbols since it provides a scenario for the interpretation and application of symbols. The research study found that symbols influenced the representation of problem situations, procedures, goals, activities, and the organization of results. This is illustrated with a linear equation problem $3x + 8 = 15$ that consisted of multiple symbols interacting with each other in the symbol system. The student's interaction with the equation revealed their ability to solve such an equation that was in accordance with the transposition method by Baiduri (2018). Each symbol in the equation was found to have a role that in its entirety, influenced the activities to be undertaken, the space to be occupied, and the organization of the working. This called for algebraic insight where algebraic expectation and linking of representation must be done in some routine (Kenney, 2007).

CONCLUSION AND RECOMMENDATION

Conclusion

The study explored students' proficiency in using mathematical symbols in secondary school in Kenya. The study findings agreed with findings from other related studies and the conceptual framework provided a lens for underpinning the study on the relationship of conception and symbol attributes to their measure of the level of proficiency. The first research question was answered by the findings that indicated learners had prior knowledge about mathematical symbols that affected their conception. In addition, the multiple meanings of symbols hampered students' understanding of concepts since most of the learners hanged on singular meanings of symbols without consideration of contexts owing to the polysemy nature of symbols. The study also reveals that students are aware of the role of mathematical symbols in secondary mathematics for conceptual understanding in areas of usefulness in solving mathematics problems owing to the clarity and succinct nature of symbols and that belief and attitude are a factor in mathematics learning. Finally, the multiple representation and problem solving were major occasions in which students used symbols in mathematics.

Recommendations

The study revealed the influence of symbol meanings and prior knowledge in the learning of mathematics and the role of symbols in mathematics learning. And ways in which symbol use manifested by students, such as multiple representations and problem-solving. The emphasis ought to be put on the meaning of symbols so as to reduce the hampering in the learning of mathematics since many textbooks place symbols without explanation about them. Also, multiple representations may be given considerable priority as it is linkage to other situations of use is indispensable. Better and more

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effective instructional strategies may be adopted to execute the teaching and learning of symbols.

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