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Original Article

A Systematic Review and Bibliometric Analysis of Probability Concepts: Research Methods and Frameworks

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Mathematics
Education.

This study presents a systematic literature review and bibliometric analysis focused on students' understanding of probability concepts, reflecting a growing interest in this area. It synthesises findings from 32 articles published between 2014 and 2023, selected through a structured search across multiple databases, including EBSCO, ERIC, SCOPUS, and Google Scholar. The analysis employed descriptive methods using Excel, examining various research approaches—qualitative, quantitative, and mixed methods—alongside diverse research designs and data collection instruments. The results indicate that most research on probability concepts originates from Indonesia, Turkey, and the United States. Additionally, several theoretical and conceptual frameworks were identified as foundational in the reviewed studies. The authors believe this review will inform the development of systematic review procedures and enhance research designs and theoretical frameworks related to teaching probability concepts, ultimately improving students' conceptual understanding in this domain. This synthesis aims to provide a comprehensive overview that can guide future research and instructional strategies in probability education.

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INTRODUCTION

Probability is the branch of mathematics that deals with the numerical description of how likely an event is to occur (Susanti *et al.*, 2023). As a fundamental concept in mathematics, probability has applications in many areas of the subject as well as in other academic disciplines and is frequently used in practical situations (Yusuf *et al.*, 2022). Knowledge in probability concepts is applied in fields such as financial mathematics, lottery, medical diagnosis, election results, sports outcomes, traffic signalling, insurance, industrial quality control, genetics, quantum mechanics and the kinetic theory of gases (Desse & Mengesha, 2024; Owusu *et al.*, 2022). Probability literacy is crucial for daily decision-making, including estimating rumour spread, reading newspapers, and making forecasts (Huang, 2022). Educational authorities in many countries recognise its importance and have incorporated it into their school curricula (Batanero *et al.*, 2016).

Despite its relevance, studies have shown that probability concepts have been found to be more challenging to comprehend and understand among students and teachers (Brodie, 2014; Herholdt & Sapiro, 2014; Galavotti, 2015; Baltaci & Evran, 2016; Memnun *et al.*, 2019; Astuti *et al.*, 2020; Begolli *et al.*, 2021; Hokor *et al.*, 2022; Yusuf *et al.*, 2022; Sani & Rosnawati, 2022). According to Yusuf *et al.* (2022), understanding probability and solving probability problems seem to be tough for students to do. Studies have shown that because the probability concepts are seen as abstract, teaching and learning can be challenging, and both teachers and students need assistance to work through the problems they present (Martin *et al.*, 2021; Firat & Gürbüz, 2022). This is because in addressing probability questions, students must simultaneously grasp the concepts of probability, the method for

solving problems, and knowledge of the likelihood that a problem will arise (Zorzos & Avgerinos, 2023; Usry *et al.*, 2016).

Based on certain essential elements of probability theory, students must be familiar with the idea of probability (Anggara *et al.*, 2018). Pupils' grasp of probability can serve as a foundation for learning more advanced mathematical ideas and applying them to real-world situations. However, pupils frequently struggle with grasping the notion of probability. According to Batanero *et al.* (2016), there can occasionally be a didactic problem when learning the concept of probability because of an explanation error about the relationship between classical probability accumulation and frequency approaches and abstract statistics. Yunarti (2014) describes it as a challenge in comprehending the notion of prerequisites. The review of prior studies has predominantly focused on problem-solving in probability topics such as joint event probability, conditional probability, and Bayesian networks (Zorzos & Avgerinos, 2023; Yusuf *et al.*, 2020). Research shows students face challenges in tackling probability problems, often overshadowed by problem-solving skills (Hendricks & Olawale, 2023). In the view of Vásquez-Ortiz and Alsina (2021), the teaching and learning of probability concepts to students presents conceptual complexity and developmental challenges.

To enhance students' understanding of probability concepts, it is crucial for them to recognise theoretical frameworks employed in research studies (Hendricks & Olawale, 2023). These frameworks aid in conceptualising and reasoning about uncertainty, informing instructional design in probability and statistics education. Additionally, students should grasp the mathematical foundations of probability theory (Sihotang & Zuhri, 2022). Limited research studies have been conducted on

the theoretical and conceptual frameworks of probability concepts, contributing to students' learning difficulties. The existing research highlights challenges faced by students in comprehending probability, such as misconceptions that persist across different grade levels, gender-based differences in misconceptions, and varying levels of difficulty based on students' abilities (Arum *et al.*, 2018). Additionally, the lack of a solid theoretical structure for early probability learning hinders curricular decisions and further research, showcasing the need for more in-depth studies in this area. According to Anggara *et al.* (2018), identifying students' struggles in understanding probability concepts is crucial for educators to tailor effective teaching strategies and address obstacles in the learning process.

Therefore, it is expected that this review will contribute to the development of systematic review methods, research designs and methodologies, theoretical and conceptual frameworks that are especially focused on the teaching and learning of probability concepts, ultimately improving students' conceptual grasp of those concepts. For this study, we conducted a systematic literature review on probability concepts, with an emphasis on empirical studies on students' conceptual knowledge and

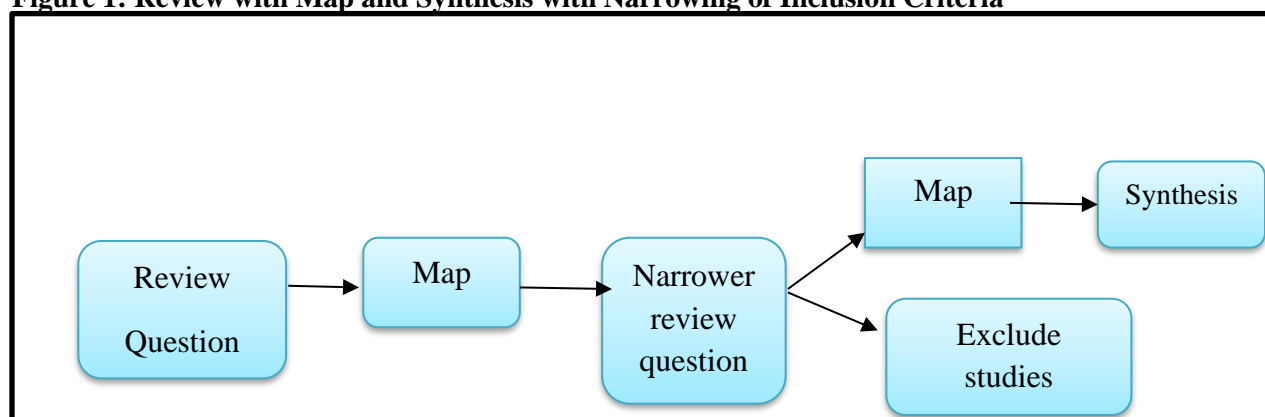
understanding. The study aimed to address the following research questions in an effort to identify trends in this field's research and future directions for investigation: (1) Which research approaches and designs are mostly employed?; (2); Which theoretical frameworks dominates studies on probability concepts and probability problem-solving?; (3) Which are the most researched counties?

METHODS

Inclusion Criterion

The conduct of successful systematic reviews is anticipated to maximise the rigour and applicability of research for policy concerns (Telep & Weisburd, 2023). This is to say that authors are expected to choose appropriate approaches that will suit the review questions and conduct the review in a way that the effectiveness of the supporting evidence should be evaluated in accordance with specific standards (Satnarine, 2023). According to the Evidence for Policy and Practice Information and Coordinating Centre (EPPI-Centre, 2016), systematic reviews are conducted using two stages. The first stage will identify, list and characterise the research that is currently available, including its focus, design, and setting of studies (see figure 1).

Figure 1: Review with Map and Synthesis with Narrowing of Inclusion Criteria



The chosen evidence was thoroughly examined and synthesised in the second stage in order to answer the research questions. For this systematic review, the authors adopted the review process as proposed

by Pickering *et al.* (2015). The systematic literature review methodology, according to Pickering *et al.* (2015), enables researchers to be methodical in the procedures they use to (i) search, (ii) survey, and

(iii) choose research papers for conducting a critical analysis of existing research bodies. Through this procedure, the authors were able to first discover the pertinent literature, then critically assess it, and last,

briefly offer a review of pertinent research. The essential steps the author took to find the relevant literature and weed out unrelated research studies are briefly summarised in this section.

Table 1: Inclusion Criteria

Criterion	Inclusion criteria
Focus	Students' difficulties and misconceptions in probability problem-solving.
Level of education	Pre-tertiary (basic and secondary) and tertiary (higher education).
Period of Research	For this study, articles published between 2014 and 2023 were considered. These are current literature that will provide the current trends in this type of research study.
Research approach	Qualitative, quantitative and mixed methods
Geographical Spread	Studies conducted across all continents.
Design	Case study, document analysis and comparative study.
Research Base	The data for the research study were based on empirical evidence.

Search Strategy

The electronic databases and institutional databases were searched for relevant literature. On the basis of the reference lists that some chosen publications supplied, more research was also conducted. This was done to ensure a wider reach and lower the possibility of bias. A structured search procedure utilising several databases, such as EBSCO Academic Search Premier, Education Resources Information Centre (ERIC), SCOPUS and Google Scholar, was employed to undertake an extensive examination of scholarly literature. Because they are the most widely used databases in the field of education, to which the study belongs, these were deemed relevant. Additionally, all of these databases were freely accessible to the authors. The Kwame Nkrumah University of Science and Technology, Kumasi, Ghana repository was one of the institutional databases that were looked through. Free text and thesaurus phrases were utilised in both electronic and manual searches to find all pertinent articles. These include 'probability', 'problem-solving', 'learning probability', 'students' difficulties in learning probability', 'misconceptions in learning probability', 'students' errors in learning probability', 'mathematics education and probability' and 'misconceptions and errors in probability problem-solving'. Each key

phrase was independently run in each database to ensure more extensive and comprehensive coverage of the comparative research on the probability problem-solving. The analysis only considered peer-reviewed publications because they are the most trustworthy sources of scientific information.

Study Selection

A total of 412 articles were found in the initial combined search of electronic and institutional databases. When the articles were exported to the Zotero open-source reference management software to manage bibliographic data and related research materials of the articles, duplicate copies of 283 articles were removed, leaving 126 articles for additional review. On the basis of the titles and abstracts, an additional study of the publications was conducted, and 81 articles were chosen for additional investigation. For the objectives of this systematic review, synthesis and meta-analysis were performed on thirty-two (32) papers.

Data Extraction Analysis

For this systematic review, the authors used a bibliometric analysis table to extract the information and data from the downloaded articles that were selected for the study. The bibliometric analysis table was created based on the inclusion criteria developed for the systematic review. The

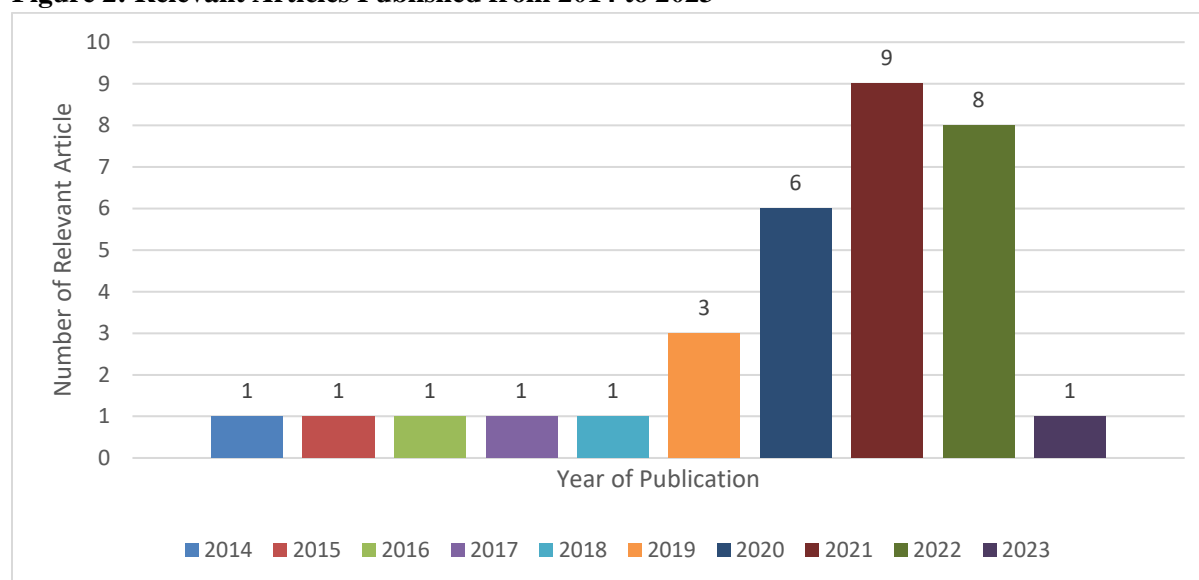
variables include the following: author(s) name, year of publication, book/chapter/article/thesis, context (study area), topic, contribution, research design, results and conclusion, and what the author(s) missed (delimitation). The research questions were used to frame the other aspects of the data that were taken from the publications. For all studies, a free-text narrative data summary was created, and content analysis was used to analyse it. The meta-analysis approach, which compiles the

findings from separate studies addressing the same issues, was used to analyse the numerical data. This method of analysis later served as the foundation for formulating findings and suggestions supported by data from all the studies mentioned.

FINDINGS

The figure below indicates the relevant articles published from 2014 to 2023 in this systematic literature review.

Figure 2: Relevant Articles Published from 2014 to 2023



Results from the findings revealed that a total of 32 relevant articles were published from the year 2014 to 2023. Of this number, it can be seen from figure 2 that 24 of the articles were published in the last 4 years, i.e., 2020 – 2023, representing 75% of the articles (for e.g., Ozkan, 2022; Kazak & Pratt, 2021; Vasquez-Ortiz & Alsina, 2021; Brückler & Milin Šipuš, 2023; Amora & Tinio, 2022; Hokor *et al.*, 2022; Awuah & Ogbonnaya, 2020; Feliciano-Semidei *et al.*, 2022; Morris, 2021; Gamze & Coşkuntuncel, 2020; Wijaya & Doorman, 2021, etc.). The remaining 8 relevant articles were published between 2014 to 2019, which also represent 25% (i.e., Salido & Dasari, 2019; Yang & Sianturi, 2019; Sepriyanti & Putri, 2018; Awuah & Folson, 2017; Beitzel & Staley, 2015; Ibrahim & Asiedu-Addo, 2019; Satake & Murray, 2015;

Baltaci, 2016). The observation from this analysis is that, there the number of relevant articles relevant to this study increased exponentially from the year 2020 to 2023. This is due to the relevance placed on probability concepts and how it's being applied in several fields such as finance, insurance, lottery, medical diagnosis, election results, genetics, quality control, sports outcomes, and traffic signalling (Desse & Mengesha, 2024; Owusu *et al.*, 2022). According to Batanero *et al.* (2016), educational authorities in many countries recognise its importance and have incorporated it into their school curricula. Probability literacy is crucial for our daily decision-making and making forecasts (Huang, 2022).

Research Approach

This systematic review categorised articles into three research approaches: quantitative, qualitative, and mixed methods. Qualitative and quantitative approaches were dominant, with 14 studies using qualitative research, 13 using quantitative research, and 5 using mixed methods. This approach allows for a comprehensive understanding of the research problem. From the perspective of the mixed method research used in the systematic review, the studies employed the combination of both qualitative and quantitative research within the same article, and this, according to Bryman (2008), facilitates a

thorough understanding of the research problem. The articles that employed the quantitative research approach mainly used numerical data, values and figures as well as illustrations in their explanation and interpretation of the research problem and findings (Muijs, 2010). The use of words rather than figures was predominantly employed in the qualitative research articles. The collection of data, numerical analysis of data and the interpretation of the results were executed using words and explanations (Hennink *et al.*, 2010) (See Table 1). Qualitative research approach (n=14 out of 32), Quantitative research approach (n=13 out of 32) and Mixed method research approach (n=5 out of 32).

Table 2: Major Research Approaches Used

Research Approach	No. of Studies	Studies
Qualitative Research	14	Salido and Dasari (2019); Ozkan (2022); Kazak and Pratt (2021); Yang and Sianturi (2019); Batista <i>et al.</i> (2022); Hadfield (2020); Sepriyanti and Putri (2018); Vasquez-Ortiz and Alsina (2021); Yusuf <i>et al.</i> (2022); Baltaci (2016); Astuti <i>et al.</i> (2020); Dayal and Sharma (2020); Brückler and Milin Šipuš (2023); Shodiqin and Sukestiyarno (2021).
Quantitative Research	13	Awuah and Folsom (2017); Ozyildirim Gumus (2021); Amora and Tinio (2022); Satake and Murray (2015); Gamze and Coşkuntuncel (2020); Wijaya and Doorman (2021); Reeves <i>et al.</i> (2021); Beitzel and Staley (2014); Milinkovic and Radovanovic (2021); Gonzalez <i>et al.</i> (2022); Begolli <i>et al.</i> (2021); Yusuf <i>et al.</i> (2020); Sani and Rosnawati (2022).
Mixed Method Research	5	Hokor <i>et al.</i> (2022); Awuah and Ogbonnaya (2020); Feliciano-Semidei <i>et al.</i> (2022); Morris (2021); Ibrahim and Asiedu-Addo (2019).
Total	32	

Research Design and Data Collection Method

This systematic review reviewed articles on various research designs and data collection methods, highlighting the importance of selecting the appropriate design for a study to obtain relevant and useful data. (Sileyew, 2019). According to Ganesha and Aithal (2022), the long-term data collection method adopted for any research must be practical, realistic, competitive and actionable. It is a

systematic, scientific, and scholarly method of gathering information from the units of analysis and samples to address the research questions of the study.

Of the five studies that employed the mixed-methods research approach, Awuah and Ogbonnaya (2020) adopted Creswell's (2015) survey research design (cross-sectional). Teacher-made achievement test (pen-and-paper) that was

explicated by the researchers was used for the data collection in their study. Feliciano-Semidei *et al.* (2022) also used a pre- and post-survey single group research design in their work. Using this research design, the researchers could examine how students' perceptions changed both before and after the instructional (teaching) module (Allen, 2017). An explanatory sequential mixed methods study design was also used by Hokor *et al.* (2022). This design "builds immediately on the results from the quantitative phase" to handle the same subject (Wisdom & Creswell, 2013). Data were gathered for this investigation in two stages. Quantitative data were gathered, and the results were analysed in the first step. In order to more accurately pinpoint preservice instructors' challenges and probabilistic misconceptions, qualitative data were gathered after quantitative data analysis. A test (quantitative) and interviews (qualitative) were used to collect data.

Morris (2021) adopted a cross-cultural research design for his study. For the collection of the data, a test was designed by the researcher for the control and experimental groups (quantitative), and semi-structured interviews were used for the qualitative analysis. The interviews were carried out in order to provide empirical evidence for the study. The research design adopted by Ibrahim and Asiedu-Addo (2019) in their study was a quasi-experimental pre-test and post-test two-group design. Tests, semi-structured interviews and questionnaires were the instruments used in the data collection.

With the quantitative research approach, where the article predominantly uses numerical data, values and figures, as well as illustrations in their explanation and interpretation of the research problem and findings, a variety of research designs are used in all the reviewed articles. The experimental research design was the one that was most frequently employed. It mainly consists of three phases: planning the experiment, designing the experiment, and doing a retrospective analysis and outcomes review (see Wijaya & Doorman,

2021; Beitzel & Staley, 2014; Milinkovic & Radovanovic, 2021; Gonzalez *et al.*, 2022). These articles adopted achievement and teacher-made tests, and in some instances interviews (Wijaya & Doorman, 2021) for their data collection.

Another research design that also dominated among the quantitative research articles was a survey research design (see Awuah & Folson, 2017; Gamze & Coşkuntuncel, 2020; Reeves *et al.*, 2021; Begolli *et al.*, 2021). The primary method for data collection in these studies was a survey, using cognitive tests and questionnaires. The researchers used a survey as the basic tool in order to gain a better understanding and comprehension of the students' (individuals and groups) perspective on probability problem-solving. The structured questions designed by the researchers in the reviewed studies were outlined in such a way that each question was used to obtain specific information for the data analysis.

Amora and Tinio (2022) and Sani and Rosnawati (2022) also used the descriptive correlation method as their research design for their studies. Diagnostic tests and a standardised questionnaire were used for the data collection. Correlation analysis was used in the data analysis. Satake and Murray (2015) employed a comparative research design in their study. The study presented a comparison of three approaches to teaching conditional probability. The researchers used an achievement test for their data collection, and this was to ascertain which of the methods of teaching conditional probability was effective for the students. The three methods approached the analysis were the formula method, the natural frequency method and the truth table method. The reviewed article by Ozyildirim and Gumus (2021) didn't have a clear research design. However, the researchers adopted a data collection tool by Estrada *et al.* (2016) as the instrument for their data collection process. The authors analysed how the level of importance that mathematics teachers place on probability and how it will be taught.

The dominant research design among the qualitative research approach was exploratory research design (Kazak & Pratt, 2021; Yang & Sianturi, 2019; Batista *et al.*, 2022; Hadfield, 2021; Sepriyanti & Putri, 2018; Brückler & Milin Šipuš, 2023). Standardised questionnaires developed by the researchers were predominantly used for the data collection. Another research design that was used in the reviewed studies was case study research design (Salido & Dasari, 2019; Yusuf *et al.*, 2022; Baltaci, 2016; Shodiqin & Sukestiyarno, 2021; Dayal & Sharma, 2020; Astuti *et al.*, 2020). The researchers in these studies also adopted interviews, tests and observations for their data collection and analysis of information. Ozkan (2022) and Vasquez and Alsina (2021) employed a study of multiple cases, which was developed by Yin (2009). This research design is descriptive and can be categorised as a case study research design. Interviews, tests and literature reviews (content analysis) were used for the collection of data.

Theoretical Framework

According to Calder and Sternthal (2023), a theoretical framework is a structure that compiles concepts and theories developed from previously tested evidence. It serves as a foundation for data analysis and interpretation of research results, and various theoretical frameworks have been utilised in mathematics education research to understand probability concepts. From the 31 reviewed articles in this systematic review, it was revealed that 11 studies were grounded on a particular theoretical framework, which were based on various theoretical frameworks that related the learning theories used in teaching and learning of probability concepts.

The theoretical framework that dominates this systematic review is constructivist theory (Hokor *et al.*, 2022; Yusuf *et al.*, 2022; Reeves *et al.*, 2021). According to Hokor *et al.* (2022), the mathematics teaching syllabus for pre-tertiary and higher education in most countries is based on the theories and principles of constructivism. Constructivism emphasises active, contextual learning, allowing

students to bring their own ideas to class. Teachers must employ this method for success, with Natalia and Kerdid (2022) stating that it focuses on developing students' understanding. The constructivism method of teaching and learning is founded on the idea that mental constructions are what cause cognition (learning). The constructivist approach is characterised by reflection, discussion, cooperation, and reciprocity. The investigations by Hokor *et al.* (2012), Reeves *et al.* (2021), and Yusuf *et al.* (2022) were grounded in constructivism because the authors were of the view that students had created some probability notions.

Another theoretical framework that also evolved in this systematic review is the Problem-Solving framework (Shodiqin & Sukestiyarno, 2021; Ibrahim & Asiedu-Addo, 2019; Salido & Dasari, 2019). These studies reviewed Polya's problem-solving technique (Polya, 1945) and Atteh *et al.*'s (2017) problem-solving framework. Understanding the problem, coming up with a plan to solve it, carrying it out, and reflecting on the situation are the four main processes of problem-solving that Polya identified. These steps have since become the framework frequently suggested for teaching and evaluating problem-solving skills (Looking back). Atteh *et al.* (2017) posit that, for the benefit of students, teachers should consequently devise and implement numerous opportunities for them to engage in problem-solving where critical thinking becomes a crucial component.

Morris (2021) also employed a theoretical framework known as the *Commognitive approach*. This discursive framework by Morris (2021) defines mathematics as a form of discourse or communication. According to Lu *et al.* (2019), this method is known as commognitive, and as the name suggests, communication and cognitive processes are seen as various manifestations of the same phenomenon. Based on the basic presumption that thinking and talking are one, the commognitive method offers a fully developed theory of "discursive" learning. This offers the operational

definitions required to enable the empirical testing of the concept of linguistic relativity. It's possible to communicate with people or with oneself without using words. It is thought to be thinking in this latter instance. Thus, the central thesis of linguistic relativity might be summarised as follows: "Language shapes thinking (discourse)" (Baikovs, 2021).

In the studies of Ozkan (2022), the researcher adopted the problem-posing design as the framework of the study. The framework which was proposed by Christou *et al.* (2015) was used to design the various tasks in the study. According to Christou *et al.* (2005), a person organises new quantitative information in response to the situation when faced with a problem-posing assignment. A problem-posing situation was divided into four cognitive quantitative processes by Christou *et al.* (2005): editing, selecting, comprehending, and translating.

It is interesting to note that the researchers in Feliciano-Semidei *et al.* (2022) employed two frameworks for their study in this systematic review: the guess-experiment-discussion approach and the game-based-instruction framework. As a framework for teaching probability, the guess-experiment-discussion approach was adopted from the studies of Castro (1998). This approach places emphasis on (a) making students' concepts clear, (b) conducting arbitrary experiments to promote cognitive conflict, (c) applying the new ideas to new circumstances, and (d) updating the prior beliefs to acquire new information. During the experiment phase, the authors of this study updated the framework by incorporating game-based education (Offenholley, 2012; Wu *et al.*, 2012).

The Bruner Theory of Representation framework was used by Milinkovic and Radovanovic (2021) in their work to determine the optimal approach for

constructing fundamental ideas in probability and statistics. In the framework, Bruner (1996) asserts that the final (internal) product of processing and coding information from the outside world is representation. No matter how complex a notion was, according to Bruner (1996), it could be successfully understood at a level that was appropriate for the child's abilities and background. He identified "images" as an alternate method of delivering outside information, which was inspired by the dual theory of knowledge communicated through action and abstraction.

Batista *et al.* (2022) adopted the framework in the studies of Bryant and Nunes (2012). In their report, Bryant and Nunes (2012) believed that probability was a highly difficult notion that required knowing a number of what are called "cognitive demands." Of these, the researchers focused on three in particular, the ones that supported their analysis:

- *Understanding randomness*: understanding the nature, consequences and use of randomness in everyday life.
- *Working out the sample space*: recognising all possible events and sequences in which they may occur; and
- *Comparing and quantifying probabilities*: understanding probability as a quantity based on proportions, and that the solution is often based on proportional calculations or relations.

The authors believe that these components were interconnected, essential to understanding probability, and required in order to address the majority of fundamental probabilistic issues.

Countries of Research

The figure below indicates the most researched countries with relevant articles in this systematic literature review, from 2014 to 2023.

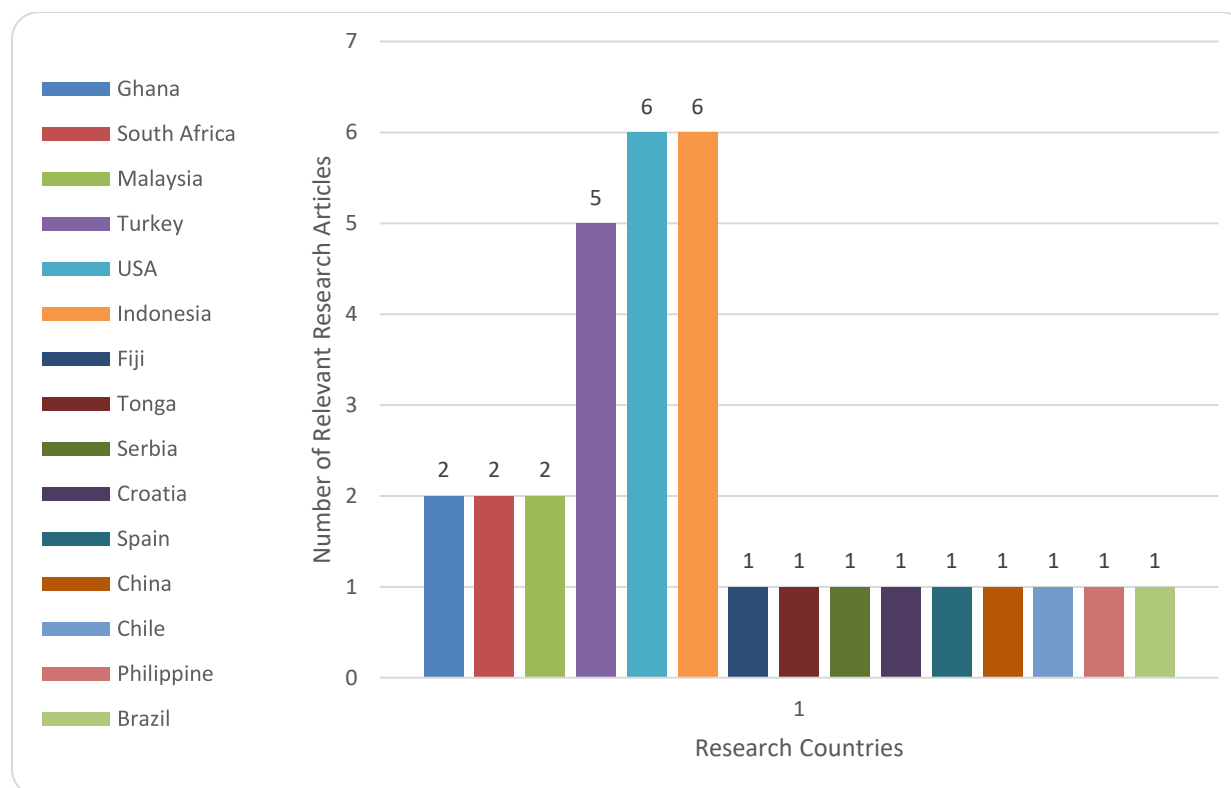
Figure 3: Researched countries with relevant articles

Figure 3 shows that, with 6 published studies apiece, the United States of America (e.g., Satake & Murray; Feliciano-Semidei *et al.*, 2022; Reeves *et al.*, 2020, etc.) and Indonesia (Salido & Dasari, 2019; Wijaya & Doorman, 2021; Sani & Rosnawati, 2022, etc.) are the two countries that have studied probability concepts the most, according to this systematic literature review. These findings could be attributed to the relevant emphasis placed on the teaching and learning of probability concepts and how they have incorporated them into their educational curriculum. Researchers in the respective countries aimed at addressing the research opportunities in the field of probability and areas it can be applied, in an effort to identify trends in this field's research and future directions for investigation. Indonesia and USA were followed by Turkey (e.g., Ozkan, 2022; Ozyildirim Gumus, 2021; Kazak & Pratt, 2021, etc.) with 5 relevant articles whiles Ghana (Hokor *et al.*, 2022; Ibrahim & Asiedu-Addo, 2021), South Africa (Awuah & Folsom, 2017; Awuah & Ogbonnaya, 2020) and

Malaysia (Yusuf *et al.*, 2020; Yusuf *et al.*, 2022) also had 2 relevant articles each to their credit in this systematic literature review. Other countries such as Philippine (Amora & Tinio, 2022), Brazil (Batista *et al.*, 2022), Chile (Vasquez-Ortiz & Alsina, 2021), China (Yang & Sianturi, 2019), Tonga (Morris, 2021), Fiji (Dayal & Sharma, 2020), Serbia (Milinkovic & Radovanovic, 2021), Croatia (Brückler & Milin Šipuš, 2023), Spain (Gonzalez *et al.*, 2022) also had one (1) relevant article each to their name in this review.

CONCLUSION

This systematic literature review presents results on 32 articles published between 2014 and 2023, analysing the most influential research approach, study designs, and complexity of research frameworks, addressing students' difficulties with probability problem-solving. The research questions focused on determining the research approach, designs, data collection methods, and the most commonly used theoretical framework in

probability problem-solving studies. Qualitative research was the most common, followed by quantitative research and mixed methods research. These findings support Sileyew's (2019) claim that research design determines data usefulness for a study to be accomplished. This systematic review used various research designs, including qualitative, quantitative, and teacher-made methods. Qualitative research used case study, experimental, exploratory, descriptive, embedded single, and multiple case study designs. Data collection tools included interviews, questionnaires, experiments, and tests. Quantitative research used descriptive-correlation, comparative, survey, experimental, and descriptive designs. Mixed method research in systematic review used exploratory sequential, quasi-experimental, cross-sectional survey, pre- and post-survey research designs, teacher-made tests, interviews, questionnaires, and experiments for data collection.

Bryman (2008) highlights the mixed method approach in systematic reviews, where studies combine qualitative and quantitative research, facilitating a full knowledge of the study subject. Quantitative research often uses numerical data, figures, and illustrations, while qualitative research uses words and explanations for data collection and interpretation. The majority of qualitative research articles utilised words instead of figures, gathering data, numerically analysing it, and interpreting results using only words and explanations (Hennink *et al.*, 2010). The systematic review found that 11 out of 32 studies used a specific theoretical framework, while 21 did not. The frameworks used included the Problem-Solving Framework, the Constructivist Learning Theory, the Commognitive Approach, the Problem-Posing Design, the Guess-Experiment-Discussion Strategy, the Game-Based Instruction Framework and the Bruner Theory of Representation (Bruner, 1996). Other authors in the systematic review also adopted theoretical frameworks developed by previous researchers, such as Polya's Problem-Solving technique (Shodiqin & Sukestiyarno, 2021; Salido & Dasari,

2019) and Atteh *et al.*'s Problem-Solving framework (Ibrahim & Asiedu-Addo, 2019), to analyze and interpret their results on probability problem-solving, providing a foundation for their research.

In conclusion, this systematic literature review has effectively achieved its objective of examining the landscape of research on probability concepts by analysing 32 articles published from 2014 to 2023. The findings reveal a clear predominance of qualitative research, followed by quantitative and mixed methods, affirming the assertion by Sileyew (2019) that research design significantly influences data applicability. By meticulously categorising research approaches, designs, and data collection methods, we have illuminated the complexities inherent in addressing students' difficulties with probability problem-solving. The review highlights the diverse theoretical frameworks employed, with 11 studies utilising specific frameworks, thereby contributing to a deeper understanding of the methodologies underpinning this field. Furthermore, the integration of qualitative insights with quantitative data, as emphasised by Bryman (2008), showcases a comprehensive approach to studying probability. Overall, this review not only synthesises existing research but also lays the groundwork for future investigations, encouraging further exploration of effective frameworks and methodologies in probability education.

Limitations of the Study

The systematic review revealed that the majority of the reviewed articles (about 20 articles) were not guided by any clear theoretical framework. This lack of a framework hinders data analysis and interpretation, as it serves as a basis and a coat hanger for the research. This is because, according to Pasque and Gilbert (2023), theoretical frameworks are crucial in research studies as they provide the foundation and theoretical support for investigations. As specialised local theories, they direct the investigation by integrating existing literature, elucidating the significance of the study,

directing the testing of predictions, and supporting the interpretation of findings (Adam *et al.*, 2023). Throughout the study process, theoretical choices are crucial and subjective, influencing the questions posed, the information acquired, the analysis carried out, and the suggestions put forth (Hiebert *et al.*, 2022). According to Calder *et al.* (2023), they play a crucial role in establishing a connection between a researcher's methodological approaches, data collection methods, analysis techniques, and trustworthiness in qualitative research. By offering a lens through which the study design is examined from beginning to end, theoretical frameworks play a crucial role in research and aid in the understanding of complicated situations (Omodan, 2022).

The systematic review mostly used qualitative and quantitative research methods, with few incorporating mixed methods, as suggested by Bryman (2008) for comprehensive research problem understanding. The systematic review of probability articles lacks technological and resource-based teaching methods, resulting in less use of physical materials in decontextualised exercises. Implementing lesson plans that consider various resources and settings could improve the probability of teaching.

Suggestions for Future Studies

Inferences drawn from the systematic review reveal that little has been done to pinpoint precisely where these learners' strengths and limitations are in terms of cognitive demand and issue material. Accordingly, identifying students' areas of strength and weakness will help them improve their problem-solving abilities. Future studies should focus on exploring students' conceptual understanding of probability concepts. Further studies can also take into consideration students' misconceptions and errors in their probability problem-solving.

Competing Interests

The authors have declared that no competing interests exist.

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