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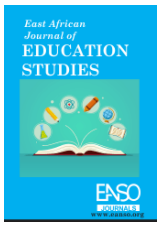

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Original Article

APOS Theory-based research in Mathematics Education: A systematic literature review

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ABSTRACT

The APOS Theory is one of the most widely used theoretical frameworks in mathematics education research, and it has enjoyed significant popularity among mathematics education researchers, especially over the last decade. In this study, we did a systematic literature review of APOS Theory-based studies published in the Scopus database over the last two decades (2005 – 2024). The aim of the study was to highlight the specific gaps in APOS Theory-based research. The results showed that a total of 125 APOS Theory-based studies were published in the Scopus database from 2005 to 2024. The trend analysis also shows that, over the last decade (2016-2024), a significant increase of an average of 12 articles per year was published as compared to an average of 3 articles per year in the previous decade (2005 – 2015). Additionally, a significant proportion of these studies (83%) were done for study groups at the university level as compared to 17% for the secondary and primary levels. The mathematical concepts in advanced undergraduate Calculus and Algebra constituted 79% of the studies. Finally, it was also found that APOS Theory was complemented with eight different theories in all the studies published. In this regard, Tall's three worlds of mathematics and the Semiotic Representation theories were the most used to complement APOS Theory. These findings bring to light, the specific gaps in APOS Theory based research, which were discussed in detail.

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INTRODUCTION

The learning of mathematics is mainly a cognitive activity. As such, the most effective theories that guide teachers in their design of instructional activities to facilitate students' learning of mathematical concepts are those that explain how learners mentally construct mathematical knowledge. Since the mission of mathematics education research is to improve the teaching and learning of mathematics, many epistemological concepts and processes in mathematics education research require theoretical treatment, without which it would be difficult to assess the scientific quality of research findings (Nilsson et al., 2018).

Among the constructivist theories of mathematical learning is Dubinsky's Action-Process-Object-Schema (APOS) Theory. Though its foundation can be traced back to the work of Jean Piaget, the fundamental ideas of this theory can be attributed to Ed. Dubinsky in the early 1980s (Arnon et al., 2014). Since then, APOS Theory has evolved to be one of the most preferred theories in most mathematics education research studies, particularly focused on mathematical learning at the university level (Oktaç et al., 2019).

The reader can refer to Arnon et al., (2014) for a detailed treatment of the APOS Theory. However, a brief overview is given here. According to APOS Theory, mathematical concepts can be learned through the mental constructions of Action, Process, Object and Schema. These mental constructions can be made through the mental mechanisms of *interiorization, encapsulation, coordination etc*.

An *Action* conception of a mathematical concept is evident when an individual can only perform an external transformation on a previously conceived mathematical object by means of an explicitly stated rule. When the individual repeatedly performs this transformation on the object and reflects on it, he begins to interiorize the action into a *Process* and gains internal control over the action. When an individual has a process conception of a mathematical concept, he or she can explain the transformations at the action stage in his or her own words without being guided by an explicitly stated rule. Applying actions to processes encapsulates the process into an *Object*. At this stage, the process is seen as a totality to which other actions can be applied. When an individual is able to coordinate all the Actions, Processes and Objects in a coherent manner in order to solve a particular mathematical problem situation, the individual is said to have developed a *Schema* for the concept Arnon et al., (2014).

At the core of APOS Theory research is the idea of a *Genetic Decomposition (GD)*. This is a theoretical model that describes the mental constructions that an individual needs to make in order to learn a particular mathematical concept. The GD is normally tested through the analysis of data which is usually collected through interviews and students' responses to test items to see if the students have made the mental constructions called for by the GD Arnon et al., (2014). Based on the data analysis, the genetic GD can be validated or revised and modified if necessary.

The power and utility of APOS Theory lies in the fact that, within the theory itself is a methodology which spells out how APOS Theory research is conducted. According to Arnon et al.(2014), the methodology involved in APOS theory-based research has three components. The first component

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is a theoretical analysis which leads to the construction of a preliminary GD of the mathematical concept to be learnt. The second component involves the design and implementation of instructional activities and tasks, based on the preliminary GD. The third component involves the collection and analysis of data from students' responses to tests and interviews. The analysis is done with the lens of APOS Theory to see if students have made the mental constructions called for by the preliminary GD. The preliminary GD is in this sense evaluated to see if it has to be discarded altogether or modified. The utility of APOS Theory in mathematics education research has manifested itself mainly in three ways.

Firstly, APOS Theory can be used as a developmental tool to facilitate the design and implementation of instruction in a way that would help students understand mathematical concepts (Breidenbach et al., 1992). Secondly, as a purely analytic and evaluation tool, APOS Theory can be used to gain insight into the mental constructions that students make in learning mathematical concepts, thereby serving as an effective tool for the assessment of students' learning (Dubinsky et al., 2013). Thirdly, in some studies, APOS Theory can be used for both purposes (Weller et al., 2011).

Because of this utility, APOS Theory has over the years received much attention in mathematics education research. Though at the very beginnings of the development of the theory, it was applied to the study of functions, it has now received much attention in the study of several advanced mathematical concepts in Calculus, Linear Algebra, Abstract Algebra, Number Theory, Set Theory, Probability and Statistics (Dogan, 2023; Fuentealba et al., 2023; Jara et al., 2019; Moru, 2020; Santos, 2019).

With the growing number of literature in the field of APOS Theory-based research, there is therefore the need to do a systematic literature review in order to identify the trends in APOS Theory-based research with the primary aim of identifying the research gaps in the field. Davies (2000) argues that systematic reviews allow both researchers and users of research to go beyond the limitations of single studies and to discover the consistencies and variability in seemingly similar studies and this in turn helps to build some degree of cumulative knowledge in educational research.

In this regard, it is therefore encouraging to note that this present study is not the first of its kind as there have been some attempts in the past by some researchers to do a systematic review of the use of APOS Theory in mathematics education. Two such studies (Bayraktar et al., 2019; ŞefiK et al., 2021) have been reported in the literature. It is also worth noting that, though these studies give us some revealing statistics that require some attention, they were silent on which concepts were the focus of the studies. Some information on this would be very useful for future research directions in specific mathematical study areas and concepts. Additionally, these studies were also silent on which other theories have been used to complement APOS Theory in the studies reviewed. Some information on this would be very useful for future research directions in investigating the teaching and learning of specific mathematical concepts and how to complement APOS Theory with other theoretical frameworks in order to further deepen the understanding of the issues investigated.

Having reviewed these systematic reviews on APOS Theory-based research in mathematics education and having identified the gaps in them, this study purposed to do another systematic review with an expanded scope to complement the past works by synthesizing and summarizing information on current trends in APOS Theory-based research. Our study was guided by the following research questions;

- What has been the trend in the number of APOS Theory-based studies published from 2005 to 2024?

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- How are the studies distributed in terms of the educational level?
- How are the studies distributed in terms of the mathematical study areas?
- How are the studies distributed in terms of mathematical concepts?
- How are the studies distributed in terms of the complementary use of other theories with APOS Theory?

METHODOLOGY

In order to answer our research questions, the systematic review methodology was used to obtain and analyze the most relevant literature on APOS Theory-based research in mathematics education from (2005-2024). Our method consists of four phases namely;

- Preliminary consideration and selection of database for searching of articles
- Comprehensive searching and downloading of articles using the appropriate keywords
- Screening downloaded articles to obtain the most relevant articles for analysis.
- Analysis, presentation, and discussion of findings

Preliminary Consideration and Selection of Databases for Searching of Articles

In our consideration of the databases for the searching of articles, the study was guided by the nature of the topic, the ease of accessibility of articles and institutional subscriptions. Since this study primarily has an educational focus, databases that are more concerned with publishing and disseminating articles in the field of education offered a comprehensive search filtering system and easy accessibility to articles were considered. The researchers of this present study were always reminded of the fact that any work on systematic review should be reproducible or replicable and as such the methodology for any good systematic review should be comprehensively clear and precise to enable its replicability by any other objective researcher. Finally, since the researchers are affiliated with a research institution, they deemed it appropriate to consider the databases that have been approved and subscribed to by the institution of their affiliation. These considerations led us to settle on the Scopus database for searching the articles for the study.

Comprehensive Search and Downloading of Articles

The Scopus database was comprehensively searched using the appropriate keyword “APOS Theory” in order to obtain the relevant articles on APOS Theory-based research.

KEY (apos AND theory) TITLE-ABS-KEY (apos AND theory) AND (LIMIT-TO (SUBJAREA , "MATH"))

This search, which was also conducted on 12th February 2025 resulted in 148 papers. These articles were further screened to obtain the most relevant for our analysis.

Screening of the Downloaded Articles

Further screening of the articles downloaded was necessary. This is because the keyword search would normally include any article that either has the word “APOS” or “Theory” in the title of the article or in the body of the article. This may include studies that are not related to mathematics. There is also the tendency for the word “APOS” to be interpreted in another language in which articles may be written which is totally unrelated to mathematics. It is also worth noting that, APOS Theory has also been widely used in other study areas in the Sciences also. So, at this phase, all the articles were screened by first reading the abstract to see which studies were actually focused on the teaching and learning of mathematical concepts. In doing this, the following inclusion and exclusion criteria were set as follows.

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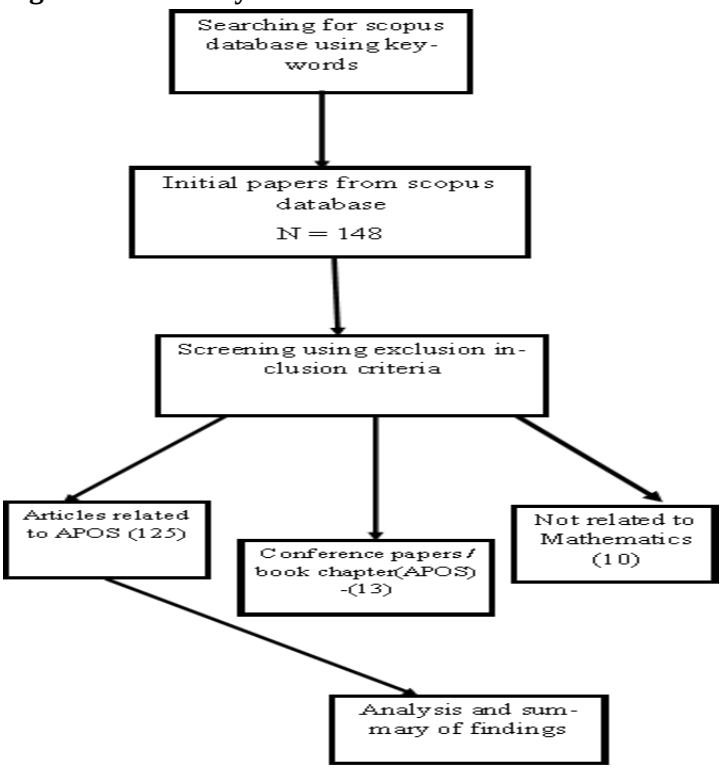
- Articles in the form of conference papers, thesis and book chapters were excluded.
- Only peer-reviewed articles that related to the use of APOS Theory either alone or together with any other theory as a theoretical lens in an empirical study in the field of mathematics education were included.

When this was done, a total of 125 most relevant articles were obtained. The chart in Fig 1. below summarizes the method.

Figure 1: Summary of Methods



Analysis

In analyzing the data, we were conscious of our research questions and this guided us to develop the framework for the analysis. A table was designed to obtain information relating to the mathematical study area and mathematical concepts for each study. Additionally, note was also of studies in which APOS Theory was complemented by or complemented another theory in the same study.

For the purpose of determining the distribution of APOS Theory-based research in terms of the mathematical study area and concepts, a broad categorization of the mathematical study areas into Calculus, Algebra/Linear Algebra, Number Theory, Set Theory, Geometry and Probability & Statistics was done. This categorization is broad enough to capture any mathematical concept that may be the focus of any mathematics-based research. The analysis was done with the main aim of describing the distribution of APOS Theory-based research in terms of the year of publication, study group, mathematical study areas and concepts and the complementary use of APOS Theory in the various studies.

RESULTS AND DISCUSSION

The results of this study are presented according to the research questions formulated at the beginning of the study.

The first research question formulated at the beginning of the study is as follows;

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What Has Been the Trend in the Number of APOS Theory-based Studies Published from 2005 to 2024?

To answer this research question, we present the Table 1.0 below

Table 1.0: Distribution of APOS Theory-based Studies from 2005 to 2024 in Scopus

Year	No. of articles	Percentage
2005	2	2%
2006	1	1%
2010	3	2%
2011	1	1%
2012	3	2%
2013	3	2%
2014	3	2%
2015	4	3%
2016	5	4%
2017	8	6%
2018	10	8%
2019	20	16%
2020	10	8%
2021	13	10%
2022	12	10%
2023	15	12%
2024	12	10%
Total	**125**	**100%**

From Table 1.0 above, the general trend shows an increase in the number of APOS Theory-based articles published per year from 2005 to 2024 in the Scopus database. Additionally, the total number of articles published in almost the last decade (2016-2024) was 105 representing 84% of all the articles published as compared to a total of 20 articles published in the previous decade (2005-2015) representing only 16% of all the articles published. Further analysis reveals that whilst the average number of articles per year in the last two decades (2005-2024) is 7, in the last decade (2016-2024) an average of 12 articles per year were published as compared to an average of 3 articles per year published in the previous decade (2005-2015). The year 2019 recorded the highest number of 20 APOS Theory-based articles published whilst the years 2006 and 2011 recorded the lowest number of 1 article each. Three conservative years 2007, 2008 and 2009 did not record any articles published.

This result shows that APOS Theory has received significant interest and attention among researchers in the field of mathematics education over the last two decades (2005-2024) and especially over the last decade (2016-2024). This may be due to the powerful utility of the theory that easily presents it as a good choice for understanding the teaching and learning of mathematics. As noted earlier, APOS Theory is one of the few theories that also has within the theory itself a research methodology for investigating the teaching and learning of mathematics and this has enabled it to enjoy such popularity among researchers in the field of mathematics education.

The second research question that was formulated at the beginning of the study is as follows;

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How are the Studies Distributed in Terms of the Educational Level?

The results in Table 2 below shows the distribution of the APOS Theory-based studies by educational level published in the Scopus database from 2005-2024.

Table 2: APOS Theory-based Studies by Educational Level

Educational level	No of articles	Percentage
University	103	82%
High school	20	16%
Both	2	2%
Total	**125**	**100%**

It is clear from the table that, most of the studies (103) representing 82% of all the studies published have focused on study groups at the university level. Additionally, 20 of the studies representing 16% of all the studies published have focused on study groups at the High School level whilst 2 studies representing 2% of all the studies published have focused on both the university and high school levels.

These findings also corroborate that of (Bayraktar et al., 2019; ŞefiK et al., 2021) which also found that most of the APOS Theory based studies have focused on study groups at the university levels leaving a significant research gap in APOS Theory-based studies at the High School (primary and secondary levels). This is however not surprising as the theory itself has its origins in the teaching and learning of advanced mathematics which is usually the focus of university mathematics education. However, the power of the utility of APOS Theory can still be explored at the lower levels of education in order to highlight some of the difficulties and challenges with the teaching and learning of mathematics from the APOS perspective.

The third research question that was formulated at the beginning of the study is as follows;

How are the Studies Distributed in Terms of the Mathematical Study Areas?

Table 3 below shows the distribution of the APOS Theory-based studies in terms of the mathematical study area published in the Scopus database from 2005-2025.

Table 3: Distribution of APOS Theory-based Studies in Terms of Mathematical Study Area

Mathematical study area	Number of articles	Percentage
Algebra/linear algebra/Abstract algebra	58	46%
Calculus/Analysis	41	33%
General	3	2%
Geometry	12	10%
Number Theory	2	2%
Probability and statistics	5	4%
Set Theory	4	3%
Total	**125**	**100%**

The results show that the mathematical study area of Algebra/linear algebra/Abstract algebra received the highest number of 58 articles representing 46% of all the articles published. This is closely followed by Calculus/Analysis which received a total of 41 articles representing 33% of all articles published. The area of Geometry received a near satisfactory number of 12 articles representing 10% of all articles published. However, the areas of Number Theory, Probability and Statistics and Set Theory

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each received significantly low numbers of 2, 5 and 4 representing 2%, 4% and 3% respectively. Additionally, 3 studies were not specific to a particular mathematical study area but consisted of general discussions of APOS Theory in developing mathematical thinking and literacy among students.

These findings again agree with similar findings from (Bayraktar et al., 2019; ŞefiK et al., 2021). The significant interest in the application of APOS theory to investigate the learning of mathematics in the field of Algebra/linear algebra/Abstract algebra and Calculus/Analysis seems to suggest that, most of the difficulties and challenges with the teaching and learning of mathematics at the university level are related to the concepts in these study areas (Dogan, 2023; Fuentealba et al., 2023; Jara et al., 2019; Moru, 2020; Santos, 2019) and hence the focus of most researchers are drawn towards these areas. Additionally, it appears that, since the origins of the theory itself is rooted in the study of advanced mathematical concepts which these study areas epitomize, the theory easily lends itself to the investigation of teaching and learning of mathematics in these areas.

The relatively low number of APOS Theory-based articles in the areas of Geometry, Probability and statistics may be due to the fact that these unique mathematical study areas themselves have a number of specific theories that have been developed by researchers for investigating the teaching and learning of concepts in these areas, as such APOS Theory may not be the obvious and most appropriate choice for investigating issues of pedagogical nature in these areas.

However, since the APOS Theory provides a general framework for the teaching and learning of mathematics, the findings of this study show that its potential has been underutilized in areas of Geometry, Probability and Statistics.

The fourth research question formulated at the beginning of the study is as follows;

How are the Studies Distributed in Terms of Mathematical Concepts?

The series of tables (Table 4 to Table 9) presented in the following sections show the distribution of the distribution of the APOS based studies according to the concepts identified in each of the seven mathematical areas in Table 3 above.

Table 4: Distribution of APOS Theory-based Studies by Algebra/Linear Algebra/Abstract Algebra Concepts.

S/N	Algebra/linear algebra/Abstract algebra	Number articles	Percentage
1	Group Theory	3	5%
2	functions/linear functions of two variables	5	9%
3	linear transformation/linear equations/inequalities/matrices	21	36%
4	Vector spaces/eigenvalues/eigenvectors	12	21%
5	Quadratic functions/equations	3	5%
6	logarithms/exponential function	3	5%
7	Complex numbers	2	3%
8	Ratio and proportion	2	3%
9	Numerical sequences	2	3%
10	Preimage, tree concept	2	3%
11	Figural pattern generalization	1	2%
12	General	2	3%
	Total	**58**	**100%**

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The result from the table above shows that most of the studies are skewed towards the linear transformation/linear equation/inequalities/matrices and Vector spaces/eigenvalues/eigenvectors concepts. These, together make up about 57% of all the studies in the mathematical study area of Algebra/linear algebra/Abstract algebra. These concepts usually form part of the advanced undergraduate mathematics courses liner Algebra/linear algebra/Abstract algebra and hence the suitability of APOS Theory in investigating the teaching and learning of these concepts. APOS Theory has been rarely used to investigate the teaching and learning of the remaining concepts which do not usually form part of the advanced undergraduate mathematics courses in Algebra/linear algebra/Abstract algebra.

Table 5: Distribution of APOS Theory-based Studies by Calculus/Analysis Concepts

S/N	Calculus/Analysis concepts	Number of articles	Percentage
1	Infinity	4	10%
2	Functions/limits	8	20%
3	series and sequence	3	7%
4	Polar coordinates	1	2%
5	differentiation/derivatives	19	46%
6	differential equations	1	2%
7	integration/integrals	5	12%
	Total	**41**	**100%**

The results show that the concepts of Functions/limits and differentiation/derivates, together constitute 66% of the APOS Theory-based studies involved in the mathematical study area of Calculus/Analysis as compared to 12% of the opposite concept of integration/integrals. This suggests that the difficulties associated with the teaching and learning of differentiation are more pervasive in undergraduate mathematics as compared to those associated with integration and hence, APOS Theory based research is skewed towards differentiation (Dogan, 2023; Fuentealba et al., 2023; Jara et al., 2019; Moru, 2020; Santos, 2019).

Table 6: Distribution of APOS Theory-based Studies by Geometry Concepts

S/N	Geometry	Number of articles	Percentage
1	Angles	1	8%
2	Area	1	8%
3	Volume	2	17%
4	right triangle	1	8%
5	Taxicab distance	1	8%
6	spatial geometry	2	17%
7	Geometric transformations	1	8%
8	fractal geometry	1	8%
9	Circle geometry	2	17%
	Total	**12**	**100%**

It can be observed from Table 6 that, there is no significant variation in the APOS Theory-based studies for the concepts in Geometry. However, the concepts of volume, spatial geometry and circle geometry, together constitute 51% of all the studies with the remaining six concepts constituting 49%.

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Table 7: Distribution of APOS Theory-based Studies by Number Theory Concepts

S/N	Number Theory	Number of articles	Percentage
1	Number pattern generalization	1	50%
2	Set of natural numbers	1	50%
	Total	**2**	**100%**

The results from Table 7 above show that, the two Number Theory concepts have received equal APOS Theory-based research studies. While Number Theory is an important field in Pure mathematics with many applications, APOS Theory has been rarely used to investigate the teaching and learning of concepts in this field.

Table 8: Distribution of APOS Theory-based Studies by Probability and Statistics Concepts

S/N	Probability and statistics	Number of articles	Percentage
1	Normal distribution	1	20%
2	Baye's theorem	1	20%
3	continuous random variable	1	20%
4	Hypothesis testing	1	20%
5	least square estimation	1	20%
	Total	**5**	**100%**

Similar to the results obtained in Table 7, it can also be seen in Table 8 that, all the five concepts in Probability and statistics have received significantly low but equal APOS Theory based research studies. Another important observation from Table 7 above is that, all the concepts investigated form part of the undergraduate introductory statistics courses. This calls for more work to be done in APOS Theory-based research in advanced undergraduate statistics.

Table 9: Distribution of APOS Theory-based Studies by Set Theory Concepts

S/N	Set Theory	Number of articles	Percentage
1	Equivalence structures on sets	1	25%
2	Cardinality	1	25%
3	Infinity	1	25%
4	Subset/Power set/intersections and unions	1	25%
	Total	**4**	**100%**

Similar to the results obtained in Table 8, it can again be seen in Table 1.8 that, all the four concepts in Set Theory have received significantly low but equal APOS Theory-based research studies.

The fifth research question that was formulated at the beginning of the study is as follows;

How are the Studies Distributed in Terms of the Complementary Use of Other Theories with APOS Theory?

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Table 10: Distribution of APOS Theory-based Studies by the Complementary Use of Other Theories with APOS Theory.

S/N	Theoretical framework(s)	Number of studies	Percentage
1	APOS	111	89%
2	APOS/Semiotic Representation Theory	3	2%
3	APOS/Tall's three worlds of mathematics	3	2%
4	APOS/OSA	3	2%
5	APOS/Advanced mathematical thinking, Phenomenology	1	1%
6	APOS/VA (visual-analytic) strategy coordination	1	1%
7	APOS/Plomp model	1	1%
8	APOS/Campos model	1	1%
9	APOS/SOLO	1	1%
	Total	**125**	**100%**

Another power and utility of APOS Theory is seen in how it easily complements other theories in investigating the teaching and learning of mathematics. The results in Table 10 above show that, APOS Theory has been used to complement eight different theories to investigate the teaching and learning of mathematics. Whilst in 89% of the studies published, APOS Theory was used alone, in 11% of the studies published, APOS Theory was used to complement other theories. The three theories that have complemented APOS Theory the most are Semiotic Representation Theory (Martínez-Planell et al., 2015, 2017; Martínez-Planell & Trigueros, 2019), Tall's three world of mathematics (Altieri & Schirmer, 2019; Radmehr, 2024; Taghizadeh Bilondi & Radmehr, 2023) and the OSA theory (Borji et al., 2018, 2020; Font Moll et al., 2016) with each constituting 2% of the studies published. The remaining five theories constituted 1% each.

In our view, these results speak to the powerful utility of APOS Theory in investigating the teaching and learning of mathematics. This suggests that, in some sense, APOS Theory is a unified theory that can be used in investigating the teaching and learning of mathematics as it presents a framework that can used to understand the learning of any mathematical concept.

CONCLUSION

The APOS Theory framework has been widely used to investigate the teaching and learning of mathematics especially over the last decade. However, whilst it has been mostly used for study groups at the university level, not much has been done for the secondary and primary levels. Additionally, the mathematical concepts derivatives and integrals in Calculus and Vector spaces/matrices/eigenvalue/eigenvectors in Algebra have received the most attention from APOS Theory-based researchers leaving a lot to be done in the areas of Geometry, Probability and Statistics, Set Theory and Number Theory. Finally, APOS Theory can be used to complement other theoretical frameworks in investigating the teaching and learning of mathematics in order to deepen the understanding of the issues being investigated.

RECOMMENDATIONS FOR FUTURE RESEARCH

It is clear from the discussions so far that APOS theory is a powerful and useful theory that can be used as an analytic framework for most studies in mathematics education. However, our findings have shown that the theory has not been sufficiently explored in the mathematical study areas of Number theory, geometry, set theory and probability & statistics. Since these study areas offer a large number of mathematical concepts whose adequate

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understanding can prepare students for studies in advanced mathematics, an application of APOS theory to understand students' difficulties in learning these concepts would be worth the effort.

One conspicuously missing application of the APOS framework from our findings is its use of mathematics textbook analysis. This is a very important and growing field in mathematics education. The complementarity of the APOS theory can further be explored in mathematics textbook analysis by combining it with the Anthropological Theory of Didactics (ATD), which happens to be one of the popular analytic frameworks in mathematics textbook analysis.

REFERENCES

Altieri, M., & Schirmer, E. (2019). Learning the concept of eigenvalues and eigenvectors: A comparative analysis of achieved concept construction in linear algebra using APOS theory among students from different educational backgrounds. *ZDM - Mathematics Education*, *51*(7), 1125–1140. Scopus. https://doi.org/10.1007/s11858-019-01074-4

Arnon, I., Cottrill, J., Dubinsky, E., Oktaç, A., Roa Fuentes, S., Trigueros, M., & Weller, K. (2014). *APOS Theory: A Framework for Research and Curriculum Development in Mathematics Education*. Springer New York. https://doi.org/10.1007/978-1-4614-7966-6

Bayraktar, F., Tutak, T., & İlhan, A. (2019). An Analysis of The Studies on The APOS Theory. *Elektronik Eğitim Bilimleri Dergisi*, *8*(16), 242–251.

Borji, V., Font, V., Alamolhodaei, H., & Sánchez, A. (2018). Application of the complementarities of two theories, APOS and OSA, for the analysis of the university students' understanding on the graph of the function and its derivative. *Eurasia Journal of Mathematics, Science and Technology Education*, *14*(6), 2301–2315. Scopus. https://doi.org/10.29333/ejmste/89514

Breidenbach, D., Dubinsky, E., Hawks, J., & Nichols, D. (1992). Development of the process conception of function. *Educational Studies in Mathematics*, *23*(3), 247–285.

Davies, P. (2000). The Relevance of Systematic Reviews to Educational Policy and Practice. *Oxford Review of Education*, *26*(3–4), 365–378. https://doi.org/10.1080/713688543

Dogan, H. (2023). Coordinated topics as transitional enablers towards higher-level conceptualisations of the range concept. *International Journal of Mathematical Education in Science and Technology*, *54*(9), 1819– 1832. Scopus. https://doi.org/10.1080/0020739X.2023.2260790

Dubinsky, E., Arnon, I., & Weller, K. (2013). Preservice Teachers' Understanding of the Relation Between a Fraction or Integer and its Decimal Expansion: The Case of 0.9^{-} and 1. *Canadian Journal of Science, Mathematics and Technology Education*, *13*, 232–258.

Font Moll, V., Trigueros, M., Badillo, E., & Rubio, N. (2016). Mathematical objects through the lens of two different theoretical perspectives: APOS and OSA. *Educational Studies in Mathematics*, *91*(1), 107–122. Scopus. https://doi.org/10.1007/s10649-015-9639-6

Fuentealba, C. E., Cárcamo, A. D., Badillo, E. R., & Sánchez-Matamoros, G. M. (2023). Analysis of errors in tasks related to the derivative concept: A look from the APOS theory (Action, Process, Object, and Schema). *Formacion Universitaria*, *16*(3), 41– 50. Scopus. https://doi.org/10.4067/S0718-50062023000300041

Jara, M. A. R., Lorca, A. M., Lorca, J. J. F. M., Saldias, P. V., & Del Valle Leo, M. (2019). Cognitive construction of the solution set of a system of linear equations with two unknowns.

Article DOI: https://doi.org/10.37284/eajes.8.2.3070

Ensenanza de Las Ciencias, *37*(1), 71–92. Scopus. https://doi.org/10.5565/rev/ensciencias.2194

Martínez-Planell, R., Gaisman, M. T., & McGee, D. (2015). On students' understanding of the differential calculus of functions of two variables. *Journal of Mathematical Behavior*, *38*, 57– 86. Scopus. https://doi.org/10.1016/j.jmathb.2015.03.003

Martínez-Planell, R., & Trigueros, M. (2019). Using cycles of research in APOS: The case of functions of two variables. *Journal of Mathematical Behavior*, *55*. Scopus. https://doi.org/10.1016/j.jmathb.2019.01.003

Moru, E. K. (2020). An APOS Analysis of University Students' Understanding of Derivatives: A Lesotho Case Study. *African Journal of Research in Mathematics, Science and Technology Education*, *24*(2), 279–292. Scopus. https://doi.org/10.1080/18117295.2020.1821500

Nilsson, P., Schindler, M., & Bakker, A. (2018). The nature and use of theories in statistics education. *International Handbook of Research in Statistics Education*, 359–386.

Oktaç, A., Trigueros, M., & Romo, A. (2019). APOS Theory: Connecting Research and Teaching. *For the Learning of Mathematics*, *39*(1), 33–37.

Radmehr, F. (2024). Learning Eigenvalues and Eigenvectors with Online YouTube Resources: A Journey in the Embodied, Proceptual-symbolic, and Formal Worlds of Mathematics. *PRIMUS*. Scopus. https://doi.org/10.1080/10511970.2024.2327330

Santos, E. M. (2019). *A look into students' conceptual understanding of the definite integral via the APOS model*. *2194*. Scopus. https://doi.org/10.1063/1.5139842

ŞefiK, Ö., Erdem Uzun, Ö., & Dost, Ş. (2021). Content Analysis of the APOS Theory Studies on Mathematics Education Conducted in Turkey and Internationally: A Meta-Synthesis Study. *Necatibey Eğitim Fakültesi Elektronik Fen ve Matematik Eğitimi Dergisi*, *15*(2), 404–428. https://doi.org/10.17522/balikesirnef.1020526

Taghizadeh Bilondi, M., & Radmehr, F. (2023). Students' mathematical thinking of the tree concept: An integration of APOS with Tall's three worlds of mathematics. *Research in Mathematics Education*. Scopus. https://doi.org/10.1080/14794802.2023.2292260

Weller, K., Arnon, I., & Dubinsky, E. (2011). Preservice teachers' understandings of the relation between a fraction or integer and its decimal expansion: Strength and stability of belief. *Canadian Journal of Science, Mathematics and Technology Education*, *11*, 129–159.

