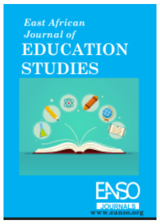

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Original Article

Analysis of Grade 11 Students' Conceptual Understanding in Probability Concepts: A Perspective of Skemp Understanding Theory

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ABSTRACT

This study investigates Grade 11 high school students' conceptual understanding of probability concepts through the framework of Skemp's understanding theory. Recognising the critical role of mathematics in everyday life and its application across various fields, this study emphasises the need for deep comprehension rather than mere memorisation. Utilising a qualitative descriptive methodology, the research analyses the scripts of eight (8) students in response to a cognitive ability test focused on probability, revealing distinct patterns of understanding categorised as relational and instrumental. The findings indicate that while some students exhibit relational understanding, demonstrating the ability to explain concepts and apply knowledge flexibly, many struggle with instrumental understanding, relying on rote memorisation and showing difficulty in problem-solving. The study identifies specific misconceptions and gaps in understanding, such as equiprobability bias, representativeness bias, belief bias and proportional reasoning. These misconceptions hinder the students' engagement with probability concepts. Ultimately, this research highlights the necessity for improved instructional strategies that promote relational understanding, fostering meaningful connections between mathematical concepts. The insights gained from this analysis aim to inform educators and curriculum developers, providing a foundation for targeted interventions that enhance students' mathematical comprehension and application in real-world scenarios.

APA CITATION

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INTRODUCTION

Mathematics is intrinsically linked to everyday life, necessitating effective problem-solving skills for students to grasp its concepts (Hsiao et al., 2017; Özreçberoğlu & Çağanağa, 2018; Phonapichat et al., 2014; Spooner et al., 2017). Understanding mathematics transcends mere memorisation; it requires a deep comprehension of material and the ability to connect problems with mathematical ideas (Cai & Ding, 2017; Helsa & Juandi, 2023). One of the primary learning objectives in mathematics education is for students to achieve a robust understanding of mathematical concepts (NCTM, 2000). This understanding is not only vital for applying mathematical principles but also for comprehending their underlying rationale.

Mathematical knowledge is seen as a dual-edged skill, facilitating both understanding and application (Cai & Ding, 2017). A strong mathematical foundation enables students to grasp concepts intuitively, impacting their overall learning experience (Chimmalee & Anupan, 2022). Understanding goes beyond simple recall; it involves integrating concepts into an internal framework (Anggraini, 2023). Kholid et al. (2021) emphasise that true comprehension allows students to engage with the material meaningfully, enhancing their ability to solve mathematical problems effectively.

According to the National Council of Teachers of Mathematics (NCTM, 2000), a comprehensive understanding of definitions, theorems, and problem-solving techniques is essential. Mathematical concepts are organised hierarchically, from the concrete to the abstract (Coles & Sinclair, 2019). Mastery of these concepts is crucial, as it lays the groundwork for learning more complex material (Skemp, 1976). However, existing research often overlooks the specific challenges students face in developing this understanding, particularly in probability concepts, which remain a poorly understood area for many. Therefore, developing mathematical conceptual understanding is vital for enhancing students' educational experiences.

Skemp's understanding theory differentiates between relational and instrumental understanding, providing educators with a framework to assess students' comprehension levels (Samosir et al., 2023). Instrumental understanding refers to the ability to use mathematical procedures without grasping their underlying principles, whereas relational understanding involves a comprehensive grasp of both "how" and "why" (Skemp, 1976). Students with relational understanding can apply knowledge flexibly, using prior experiences to tackle new challenges.

Skemp identified several characteristics of students with relational understanding, including the ability to explain concepts, adapt to new tasks, and enjoy problem-solving (Samosir et al., 2023). In contrast, students with instrumental understanding may rely on memorisation and struggle to explain their reasoning (Kaltakci-Gurel, 2023). The ability to categorise and relate mathematical concepts is crucial for developing a thorough understanding, as it fosters connections between various topics (Cai & Ding, 2017).

Understanding mathematical concepts is essential not only for academic success but also for real-world applications (Chimmalee & Anupan, 2022). Knowledge of probability concepts is crucial for various fields, including finance, insurance,

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industrial quality control, genetics, quantum mechanics, and the kinetic theory of gases (Ma, 2024; Owusu et al., 2022). Its importance has led educational authorities worldwide to integrate probability literacy into school curricula to develop future professionals (Chen et al., 2022; Batanero et al., 2016). Despite this emphasis, high school students often perform poorly and face significant learning challenges in probability, highlighting a critical need for targeted research in this area. Research indicates that many students struggle to achieve a deep and accurate understanding of basic probability concepts and problem-solving techniques (Memnun et al., 2019; Astuti et al., 2020; Begolli et al., 2021; Hokor et al., 2022; Yusuf et al., 2022; Sani & Rosnawati, 2022). Astuti et al. (2020) highlight that probability is an abstract and complex field, requiring students to grasp both concepts and appropriate problem-solving strategies to succeed. This paper aims to analyse students' conceptual understanding of probability concepts through the lens of Skemp's understanding theory, addressing these gaps and contributing to the discourse on effective mathematics education.

Literature Review

Understanding Probability Misconceptions

Skemp's theory of understanding helps educators differentiate between students who genuinely comprehend mathematical concepts and those who do not (Samosir et al., 2023). He identifies two types of understanding: instrumental and relational. Instrumental understanding refers to using mathematical processes without grasping their underlying rationale, meaning students know "how" but not "why" (Skemp, 1976). In contrast, relational understanding involves applying mathematical rules while understanding their justifications, forming essential "schemas" (Skemp, 1982). Research indicates that relational understanding is linked to concept comprehension and problem-solving, suggesting a stronger knowledge base (Samosir et al., 2023).

Traits of Understanding

Students who possess relational and instrumental understanding share a number of traits. Traits of relational understanding include: 1) being able to use "why" to explain a concept; 2) reflecting before acting; 3) providing the answer at the end; 4) adapting to any task by drawing on prior knowledge; 5) making an effort to understand; and 6) enjoying solving math problems for their own sake (Samosir et al., 2023). Accordingly, relational knowledge can enhance students' mathematics comprehension in addition to concentrating on a process to arrive at the necessary answer (Dewi & Samsudin, 2019). Conversely, students with instrumental understanding exhibit traits such as: 1) not being able to explain a concept using "why"; 2) being able to give direct answers to specific questions; 3) occasionally being unable to advance; 4) being more likely to memorize; 5) relying on the teacher's example; and 6) not finding math enjoyable (Kaltakci-Gurel, 2023).

Assessing Mathematical Understanding

Students' mathematical understanding can be assessed and analysed through Skemp's seven indicators, which include the ability to categorise objects, apply concepts algorithmically, provide examples, and correlate multiple mathematical concepts (Bakar et al., 2018). Mastering mathematical comprehension is essential, as it enables students to relate mathematical topics to broader concepts (Cai & Ding, 2017). Concept comprehension is crucial for learning, allowing students to grasp ideas that should not be retained in isolation (Chimmalee & Anupan, 2022). However, research indicates that many students struggle to meet these indicators, with none achieving the second, third, or fourth markers (Fatimah & Prabawanto, 2020). This highlights the challenges students face in developing a robust understanding of mathematical concepts, particularly in probability, which remains an area of persistent difficulty.

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Conflicts Findings and Gap

While some studies emphasise the importance of relational understanding in enhancing problem-solving skills, others suggest that students can perform adequately with instrumental understanding in certain contexts. This discrepancy underscores a gap in the literature regarding effective teaching strategies that accommodate both understanding types. Furthermore, the lack of comprehensive research on the specific misconceptions students face in probability concepts calls for targeted studies to develop more effective instructional methods. Addressing these gaps is essential for fostering deeper mathematical comprehension and improving educational outcomes.

METHOD

The research employed a qualitative methodology utilising a descriptive approach, focusing on exploring and understanding the experiences and perspectives of students in probability concepts. This method allows for in-depth insights into probability, providing a rich contextual understanding of the basic concepts. Given the exploratory nature of this study, a small sample size of 8 students was chosen to facilitate detailed qualitative analysis. This size enables a more nuanced examination of individual experiences and perspectives, which might be lost in larger samples. However, this small sample may limit the generalizability of the findings to a broader population. The analysis is based on the results of students' scripts in a cognitive ability test on probability concepts, based on Skemp's theory of understanding. Participants were selected using a purposive sampling technique, which involved specific criteria and recommendations from the authors. The participants in this study included eight (8) Grade 11 students from four different schools in Kumasi metropolis of Ghana, who read Further (Elective) Mathematics as an elective subject.

The technique for ensuring data validity is conducted in several stages, which include preparation, implementation, data collection, analysis, and report preparation. During the preparation phase, the authors in this study focused on the understanding of probability concepts and examining fundamental probability theories by the students. The primary research instrument employed in this study is a cognitive ability test on probability concepts, designed to assess both conceptual understanding and problem-solving skills. This test was derived from textbooks aligned with the curriculum and validated by experts in mathematics education.

In the data collection and analysis stage, activities involved selecting the students as research participants and administering test instruments to the students. The cognitive ability test included a variety of question types, which were open-ended, demanding short answers and problem-solving tasks, to capture a comprehensive view of students' conceptual understanding. The data was then analysed using Skemp's theory of understanding, as outlined by the authors. Subsequently, the solutions written by the students were examined in relation to the study's objectives.

The validity of the data in this study was established through data triangulation. This involves collecting data using multiple methods, such as tests, content analysis and observations. Time triangulation refers to the comparison and analysis of data to reassess the reliability of the information gathered at different times. Data analysis consists of data reduction, data presentation, and drawing conclusions, ensuring a rigorous approach to interpreting the collected data.

Data Analysis and Results

To effectively analyse the students' conceptual understanding of probability concepts through the lens of Skemp's theory of understanding, a comprehensive content analysis of their written scripts was conducted. This analytical approach was

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specifically designed to elucidate the depth and breadth of the students' grasp of the topic, allowing the authors to gain insights into their cognitive processing of probability. By aligning the assessment with Skemp's theory, which distinguishes between relational and instrumental understanding, the analysis aimed to highlight not only the correctness of the students' answers but also the underlying reasoning and conceptual frameworks they employed while solving these problems. This dual focus on both the content and the thought processes involved is crucial for identifying gaps in understanding and informing future instructional strategies that can better support students' learning in probability.

Based on the students' responses in the test, the authors analysed the different categories of understanding in probability concepts. The indicator for conceptual understanding of mathematical concepts used for this analysis are as follows: The indicators of mathematical understanding of the concepts utilized in this study are as follows: 1) students' ability to classify objects based on needs that can form a concept; 2) students' ability to apply concepts algorithmically; 3) students' ability to give examples of a concept; 4) the ability to repeat the concepts learned; 5) students' ability to provide some mathematical concepts; and 6) students' ability to correlate some mathematical concepts. A purposive sampling of the scripts of the students in the test was selected and analysed based on the questions in the cognitive ability test. For the purpose of anonymity, codes such as S1, S2, S3, T1, T2, T3, T4, etc., were used for the students instead of their real names.

Question 1:

Figure 1

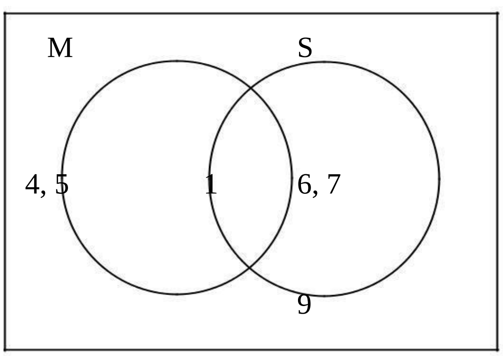
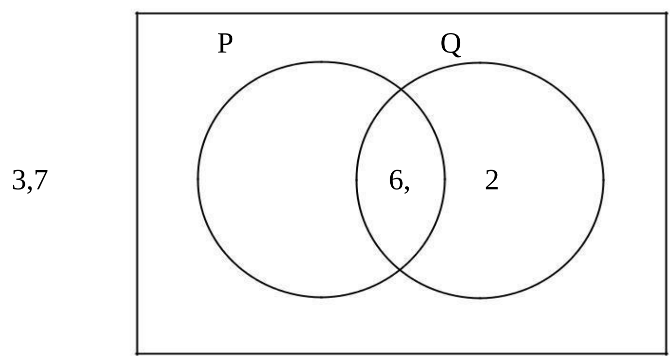


Figure 2



a) Identify with reasons the figure exhibits the underlisted properties

i) Mutually exclusive

ii) Inclusive

iii) Complementary

iv) Exhaustive

b) Write down a mathematical expression for P (A or B)

i) If A and B are mutually exclusive

ii) If A and B are independent

Answers:

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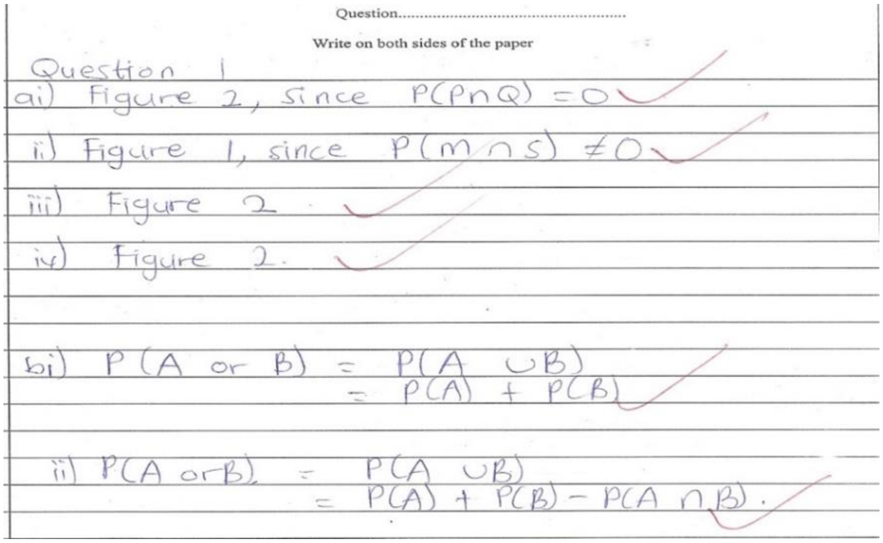
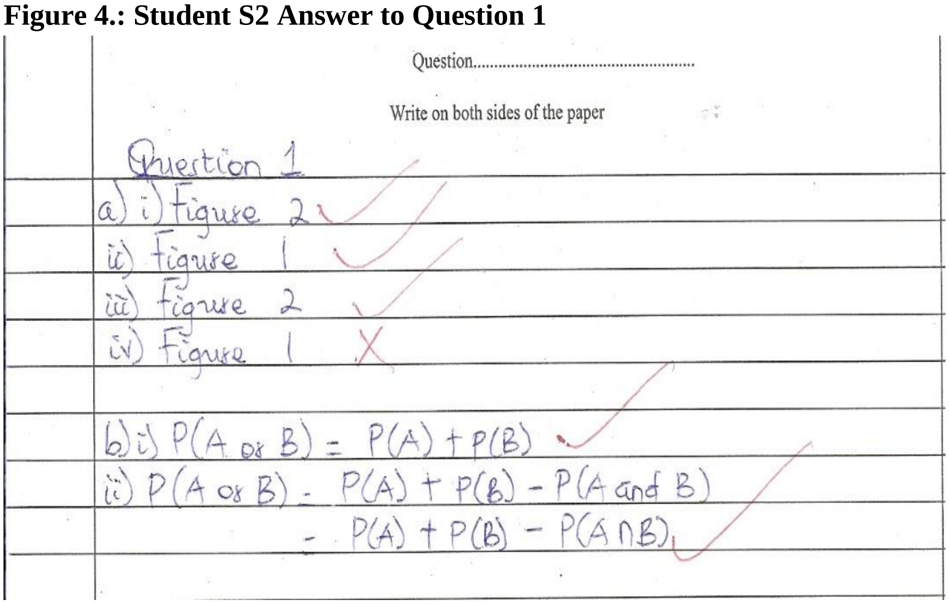
Figure 3: Student S1 Answer to Question 1



From figure 3, it can be realised that student S1 knows and understands the definition of the underlisted properties, i.e., mutually exclusive, inclusive, complementary and exhaustive. Student S1 was able to identify with reasons why the figure had the above-listed properties. Based on the discussions regarding question 1, it appeared that student S1 has a good understanding of indicators 1, 2 and 3 of the mathematical conceptual understanding according to Skemp's theory. Such a student exhibits relational understanding since the student was able to provide reasons for the answers provided for question 1a) i) - iv). This is in line with the first three characteristics of relational understanding, i.e., 1) being able to use 'why' to explain a concept; 2) reflect before acting; 3) provide the answer in the end.

Figure 4.: Student S2 Answer to Question 1



Analysing figure 4 above, student S2 wasn't able to solve all the questions in question 1. Student S2 was able to provide the needed answer for question 1a), but could not provide the reasons for the answers provided. Based on the analysis of the

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solution, student S2 appeared confused, had difficulty answering the remaining questions and didn’t remember the previous basic probability concepts they might have learnt. Again, since student S2 couldn’t give a reason for the answer provided for question 1a), the student answered the question by guessing. Based on this, student S2 can be said to have met indicator 3, which reads ‘students to provide examples of a concept’, which is in line with characteristic 3 of relational understanding according to Skemp’s theory (i.e., ‘providing the answer at the end’). However, student S2 does not meet indicator 1 because the student was not able to explain the properties using ‘why’. Student S2 was only able to provide direct answers to question 1a), specifically without any reason. The student simply relied on memorisation of the answers without any understanding of the probability concepts.

Question 2:

Given that **P(A) = 0.6**, **P(B) = 0.5** and **P (A ∩ B) = 0.2**

a) Illustrate the above information on a Venn diagram

b) Find P (A ∪ B)

c) Find P (A ∪ B)’

d) Show that events A and B are not independent.

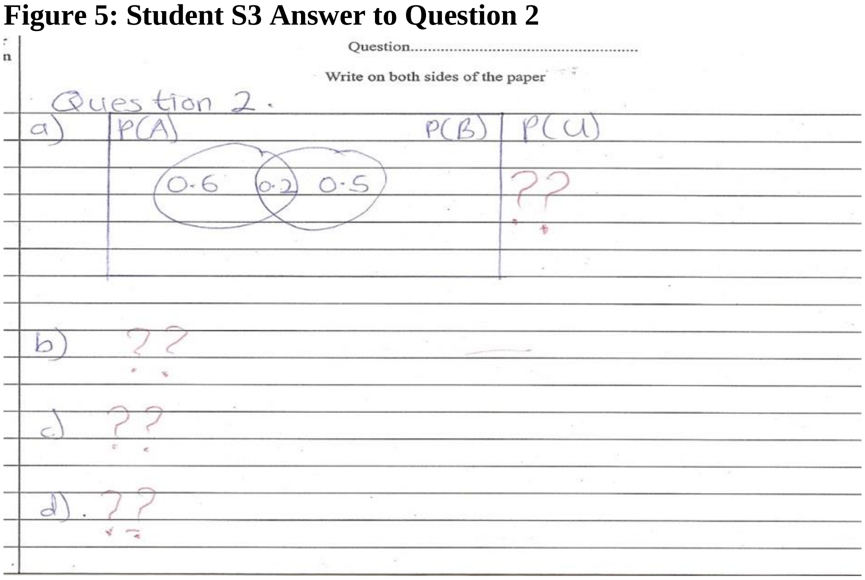
e) Are the events A and B complementary?

f) Give two reasons to support your answer in question e) above.

Answers:

For question 2, students were given a preamble where they were supposed to illustrate the information on a Venn diagram (in question 2a). This was to ascertain their level of conceptual understanding in the use of a Venn diagram as a visual aid in solving probability problems.

Figure 5: Student S3 Answer to Question 2



From Figure 5, it can be realised that student S3 doesn’t understand the purpose of the question (question 2) even though the student was able to draw the Venn diagram correctly. Student S3 subsequently was able to draw the Venn diagram, but doesn’t understand what’s being asked in question 2b‐2f. Student S3 only wrote down the information he/she knew without writing down what was being asked for in the subsequent questions. Based on the outcome of the analysis,

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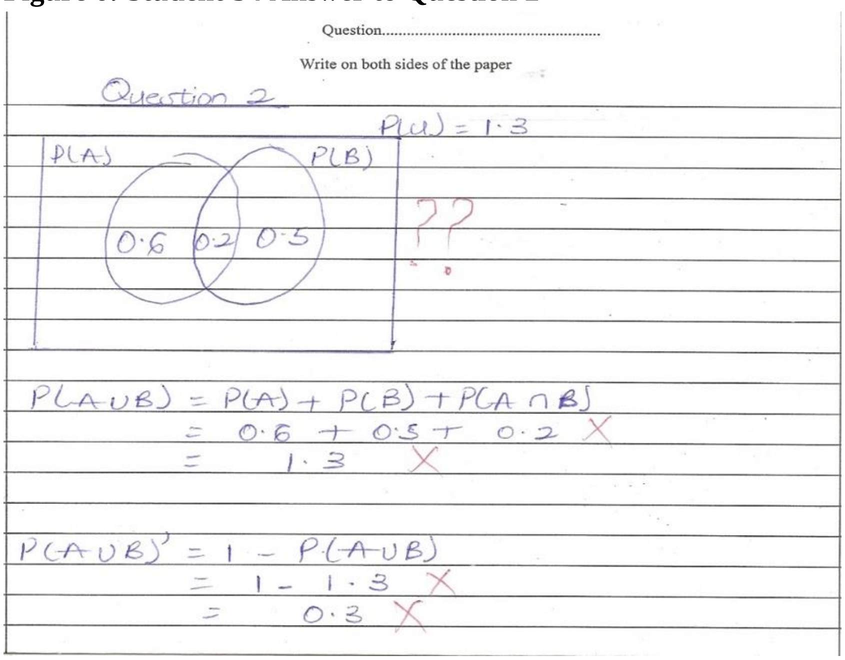
student S3 remembers writing the symbols for union, intersection and complement of a set, as applied in probability problems involving the use of Venn diagram. However, the student couldn't apply this knowledge in solving for P (A ∪ B) and P (A ∪ B)'. Such a student doesn't meet indicator 5 of the mathematical understanding theory (i.e., 'students' ability to provide some mathematical concepts'). Additionally, student S3 didn't meet indicators 4 and 6, which subsequently not able from repeating the concepts that have been studied in relation to intersection, union and complement, and prevented them from correlating some mathematical concepts. It can be seen that student S3 possesses instrumental understanding instead of relational understanding. This is because, according to indicator 3 of instrumental understanding, the student was unable to advance to solve the subsequent questions after being able to draw the Venn diagram.

Figure 6: Student S4 Answer to Question 2



From Figure 6, it can be realised that student S4 doesn't understand the basic concepts of probability at all in relation to the use of a Venn diagram. For instance, student S4 obtained a total probability of 1.3, which is an error and a wrong answer since no probability should have a total value greater than 1. This is according to the basic definition of probability, i.e.,

$P(U) = 0.6 + 0.2 + 0.5 = 1.3 \neq 1$.

The indication is that student S4 doesn't meet indicators 4, 5 and 6 of the understanding characteristics, respectively. This is in the sense that the student had not been able to repeat the probability concepts they had learnt regarding union, intersection and complement. The student was not able to provide the correct mathematical concepts regarding the basic definition of probability was unable to correlate the probability concepts to the subsequent questions in question 2. Again, such a student exhibits instrumental understanding since the student lacks the conceptual understanding of the basic probability definition. Such a student is more likely to memorise concepts, will only rely on teachers' examples and may not find studying mathematics enjoyable, as according to Kaltakci-Gurel (2023).

 |

Question 3

Mr. Emmanuel has a bag containing 5 green balls and 7 red balls. Two balls are picked at random from the bag, one after the other.

a) Illustrate the information on a tree diagram if

   i) The first ball was replaced before the second ball was picked.

   ii) The first ball was not replaced, and the second was picked.

b) Find the probability that the two balls selected were of different colours.

c) Find the probability that the two balls picked were of the same colour.

d) Find the probability that at least one of the balls picked was green.

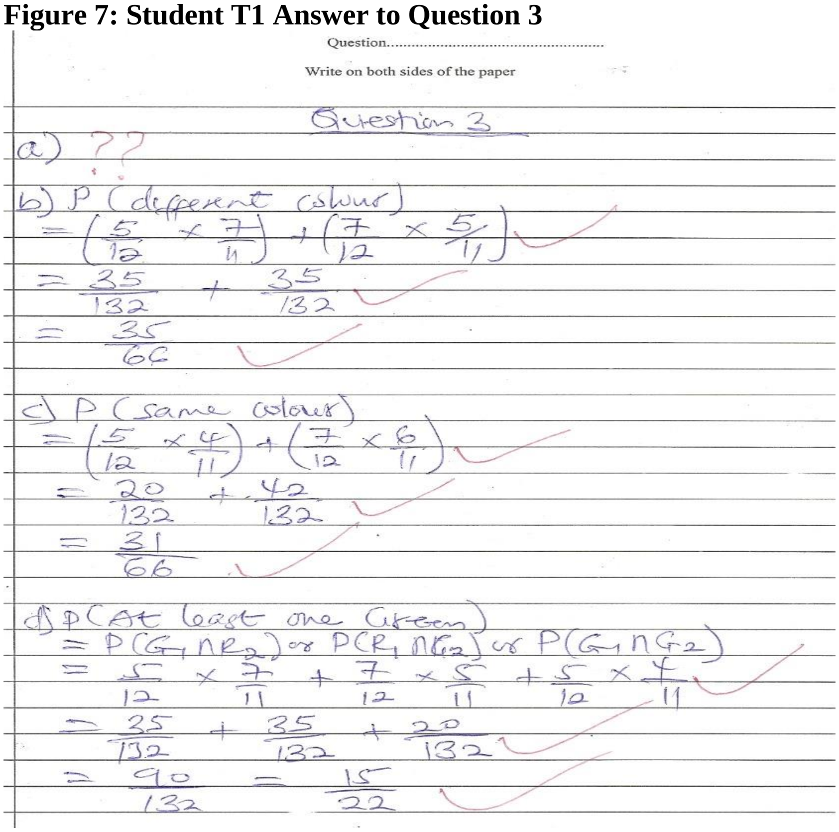
e) Besides the use of the tree diagram, provide any other way of obtaining the sample space for the question, assuming that the probability of picking a ball is independent.

f) For a number of experiments, provide any two ways by which one can determine whether a tree diagram drawn is correct or wrong.

Answers:

Analysis of students' scripts on this question was not without errors and misconceptions. Most of the students made several procedural and computational errors, even though the number of structural errors was minimal. Even though the students had an understanding of the questions, the content analysis of the students' solutions revealed that they had a challenge in drawing the tree diagram. The conceptual understanding of the students in relation to the use of a tree diagram in solving probability problems was not encouraging

Figure 7: Student T1 Answer to Question 3



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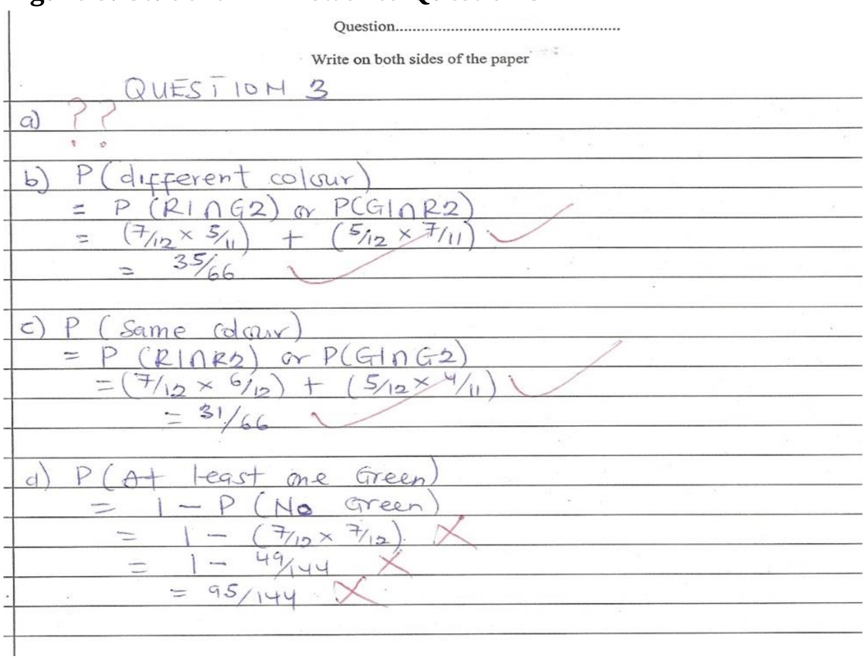
From Figure 7, it was evident that student T1 understood question 3 very well but had a challenge with drawing the tree diagram. However, student T1 was able to solve questions 3b and 3c without the use of the tree diagram. This was an indication that student T1 had met indicator 2 of the understanding characteristics, which suggests that the student was able to apply the concepts algorithmically. Nonetheless, other students couldn’t solve questions 3b and 3c. Additionally, another notable misconception identified was regarding the concepts of ‘selection with replacement’ and ‘selection without replacement’. This misconception was evident with the sample spaces they created, as they repeatedly selected from the bag containing the items. The probabilities indicated by their branches were influenced by this confusion.

The students were found to have a misunderstanding regarding the concepts of several experiments conducted and the various options from which they needed to choose an object. The analysis further revealed that the students did not grasp that the experiments conducted symbolised the number of different tree diagrams, while the branches represented the various choices available for selection at any given moment.

Figure 8: Student T2 Answer to Question 3



Again, in an attempt to solve question 3d, student T2 had the solution wrong because the student didn’t understand the phrase ‘at least one’, as framed in the question. Student T2 had been able to question 3b and 3c, similar to the solutions provided by student T1, as seen in Figure 8. In this situation, student T2 did not meet indicator 5 of the mathematical understanding characteristic. This is in the sense that the student was not able to provide the correct mathematical concept needed to solve the question. According to Skemp’s theory, such a student possesses instrumental understanding because, according to indicators 2 and 3, the student was not able to provide direct answers and was unable to make progress on the question.

Further analysis also revealed that other students faced challenges in distinguishing between ‘or’, ‘and’ and ‘at least one’. There was a noticeable lack of basic knowledge that the sum of probabilities of each branch should be equal to 1. This suggests that if these key concepts had been

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adequately explained and appropriately treated, students' conceptual understanding would have significantly improved when solving problems related to tree diagrams in probability concepts.

Question 4

Each of the 200 employees of a company took a competency test. The results are identical in the table below.

	Pass	Fail	Total
Male	A	32	D
Female	72	50	122
Total	118	B	C

a) Find the values of A, B, C and D.

b) Are the events Pass and Fail mutually exclusive?

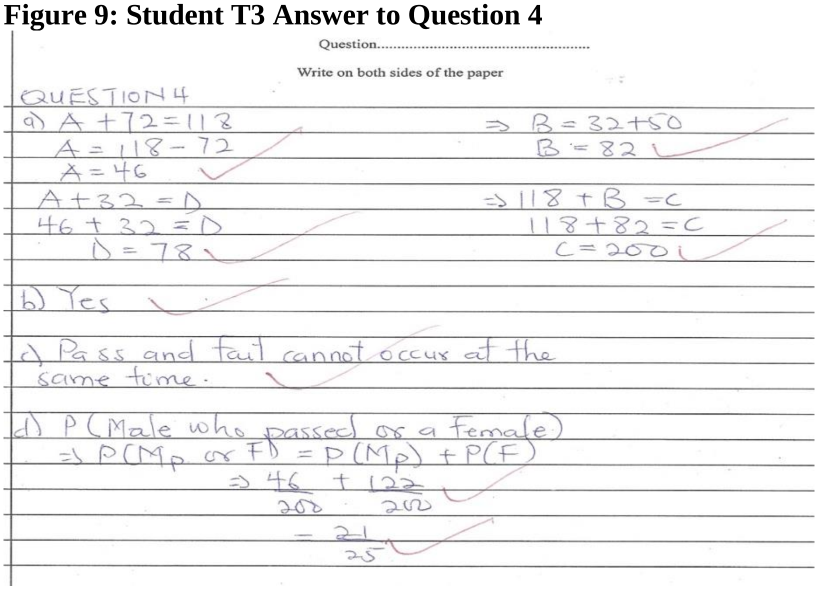
c) Explain your answer in b) above.

d) Calculate the probability that a student selected at random was a male who passed or a female.

Answers

The performance of the students in answering question 4 was very encouraging, as many attempted and answered it correctly. Comparatively, students who attempted question 4 performed better than those who attempted question 3, i.e., the tree diagram. The general assessment of these concepts was that the students felt comfortable with the questions since they involved sums (addition) and differences (subtraction), which are basic mathematical operations. The students acknowledged that contingency tables provide a clear and structured way to visualise relationships between variables and joint probabilities. They were of the view that, for problems involving multiple events, contingency tables can simplify the complexity by allowing them to see all possible outcomes. But in contrast, they claim that tree diagrams can become cluttered and hard to follow with multiple branches.

Figure 9: Student T3 Answer to Question 4


Question...
Write on both sides of the paper
QUESTION 4
a) A + 72 = 118 => B = 32+50
A = 118 − 72 B = 82
A = 46
A + 32 = D => 118 + B = C
46 + 32 = D 118 + 82 = C
D = 78 C = 200
b) Yes
c) Pass and fail cannot occur at the same time.
d) P(Male who passed or a Female)
=> P(Mp or F) = P(Mp) + P(F)
=> 46/200 + 122/200
= 21/25

From Figure 9, student T3 understands the concepts of sum and difference in relation to finding the values of A, B, C and D. Student T2 was able to determine the values of A, B, C and D using the total frequencies of each column under question 3a. Again, student T3 was able to give reasons why the

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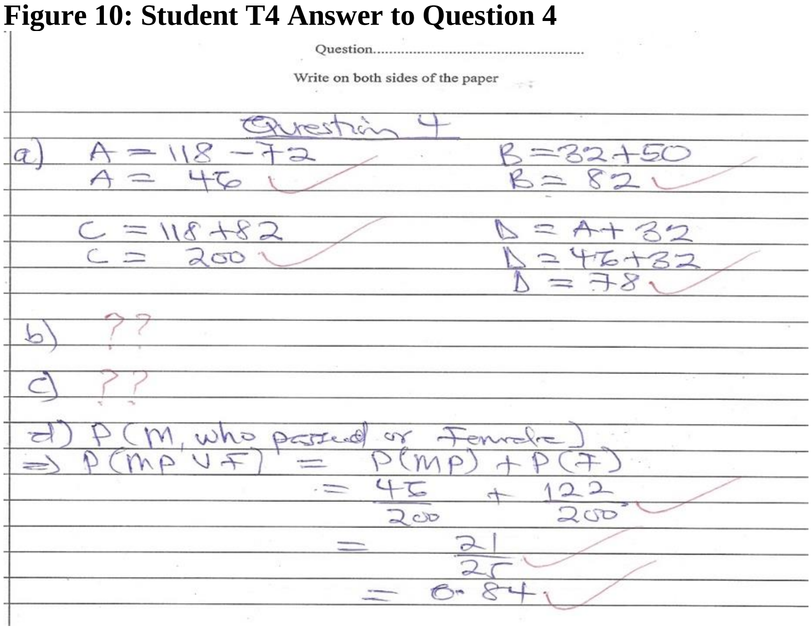
events ‘Pass’ and ‘Failure’ are ‘mutually exclusive’, i.e., $P(P \cap F) = 0$. The student concluded that the events ‘Pass’ and ‘Failure’ cannot occur at the same time. The student was able to solve the last question (question 4d) correctly without any errors or misconceptions.

Based on the analysis, it can be ascertained that student T3 had met all the indicators according to Skemp’s theory of understanding outlined in this study. Analysis of this finding suggests that such a student possesses relational understanding. The performance of student T3 in this scenario indicates that the student does not lack conceptual understanding of the basic probability concepts. According to the characteristics of relational understanding, the student was able to use ‘why’ to explain the concept of mutually exclusive events and provided the needed answers as well. Again, such a student will be able to adapt to any task by relying on the previous knowledge, which will make them enjoy solving mathematical problems in. future. This is in line with an assertion by Dewi and Samsudin (2019), which states that relational knowledge and understanding can enhance students’ mathematics understanding in addition to concentrating on a process to arrive at the necessary answer.

Figure 10: Student T4 Answer to Question 4



In the case of student T4, the first part of question 4 (question 4a) was solved correctly, similar to that of student T3. However, student T4 couldn’t explain the concept of mutually exclusive events by not attempting question 4c (as seen in Figure 10). The student didn’t understand or remember the concept of a mutually exclusive event. The indication here is that student T4 had met indicator 2 of the understanding characteristics, which reads ‘students’ ability to apply concepts algorithmically’ by being able to solve for the values of A, B, C and D in question 4a. On the other hand, the student couldn’t meet indicator 5 since the student was not able to provide the mathematical concepts in relation to mutually exclusive events. It can be concluded that such a student possesses instrumental understanding since the student was not able to explain the concepts of mutually exclusive events (i.e., not being able to explain a concept using ‘why’). In the view of Kaltakci-Gurel (2023), students who are unable to understand and explain mathematical concepts are more likely to memorise the concepts, rely on their teachers’ examples and may not find mathematics enjoyable.

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In analysing the cognitive ability test scripts from the students in this content analysis, it became apparent that not all indicators of mathematical conceptual understanding regarding probability concepts were met. The students' solutions revealed errors and misconceptions. This evidence aligns with Skemp's theory of understanding, indicating that students struggled to fulfil the criteria for mathematical conceptual understanding in the topic of probability. These observations pointed to specific deficiencies in the students' grasp of essential probability concepts, suggesting that their mathematical knowledge in this area was either insufficient or underdeveloped. As a result, this finding prompts important questions about the effectiveness of the teaching methods employed for probability and highlights the necessity for targeted interventions to improve student understanding in this field.

DISCUSSION

Relational understanding involves a comprehensive grasp of concepts, allowing students to explain "why" a solution works. For instance, student S1 demonstrates strong relational understanding by correctly identifying and reasoning through properties such as mutually exclusive, inclusive, complementary, and exhaustive. This ability aligns with Skemp's assertion that relational understanding enhances problem-solving and deepens memory retention (Wang & Yang, 2018). This is echoed by Hsiao et al. (2017) and Siregar et al. (2023), who emphasise that understanding mathematical principles must go beyond rote memorisation, thereby reinforcing the importance of relational understanding in mathematics education. Such students can adapt their knowledge to various contexts, indicating a solid understanding of probability concepts. Conversely, students exhibiting instrumental understanding rely on memorisation and might provide correct answers without comprehension. Student S2, for example, was able to give answers for question 1a) but failed to explain them, relying on rote memorisation. This failure highlights a lack of relational understanding, as student S2 could not provide reasons for their answers, reflecting Skemp's characterisation of instrumental understanding. Such students often rely on rote learning and memorisation and demonstrate a lack of enjoyment or engagement in mathematics, which can hinder their overall academic success (Kaltakci-Gurel, 2023).

The results from the data analysis reveal that students' performances vary significantly concerning Skemp's indicators of understanding. For example, student S1 exemplifies relational understanding, as evidenced by their ability to articulate reasons for identifying properties such as mutually exclusive and inclusive, which aligns with Skemp's indicators of explaining "why" (Indicator 1) and reflecting before acting (Indicator 2). However, it is important to acknowledge the limitations of this study, particularly the small sample size of 8 students, which may not adequately represent the broader student population. Additionally, potential cultural biases inherent in Ghanaian schools could influence students' engagement with mathematical concepts, as educational practices and societal attitudes towards mathematics may vary widely across different cultures.

Such students not only perform well but also demonstrate an intrinsic motivation to understand mathematical concepts. Similarly, student T3 further supports this notion by showing a clear understanding of mutually exclusive events and successfully solving complex problems without errors. Their ability to provide justifications indicates a strong relational grasp of probability concepts, affirming that relational understanding fosters deeper learning and retention. In contrast, Student S2 and Student S4 demonstrate instrumental understanding. Student S2 managed to answer some questions but failed to provide explanations, relying primarily on memorisation, which reflects Skemp's definition of instrumental understanding as a lack of conceptual clarity that

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allows the student to follow procedures but struggles to make connections between concepts. Similarly, Student S4's inability to correctly calculate a total probability and misunderstanding of fundamental concepts further illustrate the limitations of instrumental understanding. Such students often rely on rote learning and exhibit a lack of enjoyment or engagement in mathematics, which can hinder their overall academic success (Kaltakci-Gurel, 2023). These findings highlight the necessity of integrating relational understanding into teacher training programs, equipping educators with strategies to foster deeper engagement and comprehension among students.

The analysis also highlights significant misconceptions among students, particularly regarding basic probability concepts. For instance, student S4's calculation of a total probability exceeding 1 indicates a fundamental misunderstanding of probability principles. Such errors further reinforce the necessity of relational understanding, as students without a solid grasp of underlying concepts are more prone to making procedural errors. Another example was the confusion surrounding "selection with replacement" and "selection without replacement", which indicates a deeper issue with conceptual understanding. When students cannot differentiate between these concepts, it impacts their ability to solve problems effectively, as seen in students' attempts to draw tree diagrams, which has been echoed in a study by Das et al. (2022). Addressing these misconceptions through targeted teaching strategies could significantly enhance students' conceptual understanding and inform curriculum changes that prioritise relational understanding in mathematics (Chimmalee & Anupan, 2022).

Interestingly, students performed better on question 4 than on question 3, which involved tree diagrams. This disparity suggests that students may feel more comfortable with basic arithmetic operations such as addition and subtraction, compared to more abstract representations of probability. The positive reception of contingency tables indicates a preference for structured visual aids over more complex diagrams, which may be perceived as cluttered and confusing. The findings illustrate a spectrum of understanding among senior high school students regarding probability concepts, as framed by Skemp's theory. While some students, like S1 and T3, exhibit relational understanding and can explain concepts comprehensively, others, such as S2 and S4, struggle with foundational knowledge, relying instead on memorisation. This analysis underscores the importance of fostering relational understanding in mathematics education, as it not only enhances problem-solving capabilities but also nurtures a genuine appreciation for the subject. By identifying specific misconceptions and providing targeted instructional support, educators can cultivate a deeper conceptual understanding of probability among students, ultimately leading to improved academic outcomes.

CONCLUSION

This study aimed to analyse Grade 11 high school students' conceptual understanding of probability concepts through the lens of Skemp's understanding theory. The findings reveal significant insights into students' grasp of mathematical principles, highlighting a distinction between relational and instrumental understanding. While some students demonstrated a robust relational understanding, characterised by the ability to explain concepts, apply knowledge flexibly, and solve problems effectively, many exhibited instrumental understanding, relying heavily on memorisation without a deep comprehension of underlying principles. The errors and misconceptions identified in students' responses indicate a critical need for enhanced instructional strategies in teaching probability. To address these challenges, educators should consider implementing specific pedagogical strategies, such as visual aids, like concept maps and diagrams, to help students visualise relationships between concepts. Additionally, adopting problem-based learning approaches can encourage students to engage with real-world scenarios, fostering deeper connections to the material and promoting relational understanding. This research underscores the importance of integrating pedagogical approaches that foster relational understanding, encouraging students to connect mathematical concepts meaningfully. In conclusion, addressing the gaps in students' understanding of probability is essential for their overall mathematical development. Future research should focus on conducting longitudinal studies to track the progression of students' understanding over time, as well as utilising larger sample sizes to enhance the generalizability of findings. Implementing targeted interventions and refining teaching methodologies will be crucial to improving conceptual comprehension in mathematics education. By doing so, educators can better equip students to navigate the complexities of probability and enhance their problem-solving skills, ultimately fostering a more profound appreciation for mathematics in real-world contexts.

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Competing Interests

The authors have declared that no competing interests exist.

REFERENCES

Anggraini, S. (2023). Learning Concepts Learning Approach Models in Improving Students' Understanding of Mathematical Concepts. *EDUCTUM: Journal Research*, *2*(5), 1-4.

Astuti, D., Anggraeni, L., & Setyawan, F. (2020). Mathematical probability: Student's misconception in higher education. In *Journal of Physics: Conference Series* (Vol. 1613, No. 1, p. 012009). IOP Publishing.

Bakar, M. T., Suryadi, D., Tonra, W. S., & Noto, M. S. (2018). The association between conceptual understanding and reasoning ability in mathematics: An analysis of DNR-based instruction models. In *Journal of Physics: Conference Series* (Vol. 1088, No. 1, p. 012107). IOP Publishing.

Batanero, C., Chernoff, E. J., Engel, J., Lee, H. S., & Sánchez, E. (2016). *Research on teaching and learning probability* (p. 33). Springer Nature.

Begolli, K. N., Dai, T., McGinn, K. M., & Booth, J. L. (2021). Could probability be out of proportion? Self-explanation and example-based practice help students with lower proportional reasoning skills learn probability. *Instructional Science*, *49*, 441-473.

Cai, J., & Ding, M. (2017). On mathematical understanding: perspectives of experienced Chinese mathematics teachers. *Journal of Mathematics Teacher Education*, *20*, 5-29.

Chen, J., Liu, Y., Sun, Y., & Zhang, Y. (2022). Fundamental results in the probability theory. In *2nd International Conference on Applied Mathematics, Modelling, and Intelligent Computing (CAMMIC 2022)* (Vol. 12259, pp. 99-112). SPIE.

Chimmalee, B., & Anupan, A. (2022). Enhancement of Mathematical Conceptual Understanding in a Cloud Learning Environment for Undergraduate Students. *Int. J. Eng. Pedagog.*, *12*(6), 50-69.

Article DOI: https://doi.org/10.37284/eajes.8.2.2987

Coles, A., & Sinclair, N. (2019). Re-thinking 'concrete to abstract 'in mathematics education: Towards the use of symbolically structured environments. *Canadian Journal of Science, Mathematics and Technology Education*, *19*, 465-480.

Das, B. K., Jha, D. N., Sahu, S. K., Yadav, A. K., Raman, R. K., & Kartikeyan, M. (2022). Concept of Sampling Methodologies and Their Applications. In *Concept Building in Fisheries Data Analysis* (pp. 17-40). Springer Nature Singapore.

Dewi, E., & Samsudin, A. (2019). An investigation of conceptual understanding ability K-11 student of linear motion. In *Journal of Physics: Conference Series* (Vol. 1280, No. 5, p. 052055). IOP Publishing.

Helsa, Y., & Juandi, D. (2023). TPACK-based hybrid learning model design for computational thinking skills achievement in mathematics. *Journal on Mathematics Education*, *14*(2), 225-252.

Hokor, E. K., Apawu, J., Owusu-Ansah, N. A., & Agormor, S. (2022). Preservice Teachers' Misconceptions in Solving Probabilistic Problems. *Pedagogical Research*, *7*(1), 1 – 16.

Hsiao, H. S., Lin, C. Y., & Peng, Y. F. (2017). The influence of a mathematics problem-solving training system on first-year middle school students. *Eurasia Journal of Mathematics, Science and Technology Education*, *14*(1), 77-93.

Kaltakci-Gurel, D. (2023). Exploring pre-service teachers' conceptual understanding and confidence in geometrical optics: A focus on gender and prior course achievement. *Education Sciences*, *13*(5), 452.

Kholid, M. N., Imawati, A., Swastika, A., Maharani, S., & Pradana, L. N. (2021). How are Students' Conceptual Understanding for Solving Mathematical Problem? In *Journal of Physics: Conference Series* (Vol. 1776, No. 1, p. 012018). IOP Publishing.

Ma, M. (2024). Analysis of the application of probability in the field of economy. *Theoretical and Natural Science*, *36*, 220-224.

Memnun, D. S., Ozbilen, O., & Dinc, E. (2019). A Qualitative Research on the Difficulties and Failures about Probability Concepts of High School Students. *Journal of Educational Issues*, *5*(1), 1-19.

NCTM, P. (2000). Standards for School Math. *National Council of Teachers of Math*.

Owusu, F., Afutu, R., Assan-Donkoh, I., Atepor, S., & Kwakye, D. O. (2022). Use of probability theory and its perspectives. *Asian J Probab Stat*, 1-5.

Özreçberoğlu, N., & Çağanağa, Ç. K. (2018). Making it count: Strategies for improving problem-solving skills in mathematics for students and teachers' classroom management. *Eurasia Journal of Mathematics, Science and Technology Education*, *14*(4), 1253-1261.

Phonapichat, P., Wongwanich, S., & Sujiva, S. (2014). An analysis of elementary school students' difficulties in mathematical problem solving. *Procedia-social and behavioural sciences*, *116*, 3169-3174.

Samosir, C. M., Dahlan, J. A., Herman, T., & Prabawanto, S. (2023). Exploring Students' Mathematical Understanding according to Skemp's Theory in Solving Statistical Problems. *Jurnal Didaktik Matematika*, *10*(2), 219-236.

Sani, D. N., & Rosnawati, R. (2022). Students' Error Analysis In Solving Probability Story Questions Based On Revised Newman Theorem. *Journal of Mathematics Education (JUPITEK)*, *5*(2), 123-131.

Article DOI: https://doi.org/10.37284/eajes.8.2.2987

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics teaching*, *77*(1), 20-26.

Spooner, F., Saunders, A., Root, J., & Brosh, C. (2017). Promoting access to common core mathematics for students with severe disabilities through mathematical problem solving. *Research and Practice for Persons with Severe Disabilities*, *42*(3), 171-186.

Wang, K., & Yang, Z. (2018). The research on teaching of mathematical understanding in China. *American Journal of Education and Learning*, *3*(2), 93-99.

Yusuf, M., Rahim, S. S. A., & Eu, L. K. (2022). Challenges faced by college students in solving probability of event problems. *MOJES: Malaysian Online Journal of Educational Sciences*, *10*(3), 13-19.

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