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Original Article

The Exploration of Pre-Service Mathematics Teachers' Pedagogical Content Knowledge in Teaching Geometry to Primary Six Learners in Uganda

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ABSTRACT

This study explores using Pedagogical Content Knowledge (PCK) by pre-service mathematics teachers in teaching geometry to a primary six class in Eastern Uganda. The study employed a qualitative research approach using participant observations, interviews, and a documentary checklist to examine how pre-service teachers utilize examples, illustrations, explanations, demonstrations, and analogies to enhance learners' understanding of geometric concepts. The research also investigates the strategies these teachers use to address learner misconceptions and engage learners in meaningful learning experiences. Findings reveal that pre-service teachers effectively use examples and illustrations to relate abstract geometry concepts to real-life situations, facilitating learners' comprehension. However, challenges were noted in the use of analogies and in addressing learner misconceptions, highlighting areas where further pedagogical development is needed. The study also underscores the importance of rehearsal in successfully delivering classroom demonstrations. The implications of these findings suggest that teacher training programs should emphasize developing a comprehensive understanding of PCK, particularly in using analogies and correcting misconceptions. Furthermore, the study recommends enhanced support for pre-service teachers in linking geometry instruction to everyday applications, which could foster improved problem-solving skills among learners. This research contributes to the existing literature by providing insights into pre-service teachers' specific challenges and successes in applying PCK to teaching geometry, offering recommendations for curriculum development and future research in teacher education.

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INTRODUCTION

Pedagogical Content Knowledge (PCK) has emerged as a critical factor in the effective teaching of mathematics, particularly in complex subjects such as geometry. The Van Hiele model (1986) and Vojkuvkova (2012) emphasize that students progress through structured levels of geometric understanding, necessitating instructional approaches that align with their cognitive development. This structured progression underscores the importance of Pedagogical Content Knowledge (PCK), which enables teachers to tailor their instruction to facilitate students' advancement through these stages. Shulman (1986, 1987) conceptualizes PCK as integrating content expertise and effective pedagogical strategies, ensuring that complex mathematical ideas, such as those in geometry, are made accessible to learners. Therefore, examining how pre-service teachers acquire and apply PCK in geometry instruction is fundamental to enhancing teaching effectiveness and student comprehension. According to Jones & Tzekaki (2016), Geometry is a fundamental component of the mathematics curriculum. It is crucial for developing learners' spatial reasoning and problem-solving skills, essential for success in Science, Technology, Engineering, and Mathematics (STEM) fields. The significance of PCK in teaching geometry cannot be overstated, especially in the context of Ugandan education, where challenges in both content knowledge and pedagogical skills are prevalent among pre-service teachers (Komakech & Osuu, 2014; Lauwerier & Akkari, 2015; Nelson & Karlsson, 2019). Addressing these challenges through enhanced PCK development is vital for improving the quality of geometry education and preparing students for success in STEM fields.

Despite the acknowledged importance of geometry, its teaching presents unique challenges. Research indicates that many pre-service teachers struggle with content knowledge and the pedagogical skills necessary to teach geometry effectively (Walshaw, 2012). This issue is compounded in the Ugandan educational context, where Initial Teacher Education (ITE) programs often emphasize subject matter knowledge at the expense of pedagogical content knowledge (Akyeampong, Lussier, Pryor, & Westbrook, 2003). As a result, novice teachers frequently enter the classroom with a limited ability to translate their mathematical knowledge into effective teaching practices. Effective geometry instruction requires a deep understanding of the subject and the ability to anticipate and address common misconceptions, provide clear explanations, and use appropriate examples and analogies (Shulman, 1987; Shulman, 2013; Aydeniz & Kirbulut, 2014). However, pre-service teachers often lack the necessary training to develop these skills, leading to difficulties in teaching geometry concepts in a way that resonates with learners.

Uganda's educational system faces several challenges, particularly in teaching mathematics. Geometry, as a subject, is often neglected, with teachers struggling to develop the necessary competencies to teach it effectively (Nelson & Karlsson, 2019). This issue is particularly pronounced in primary six, a critical year in the Ugandan education system, as it prepares learners for the Primary Leaving Examinations (PLE) (Ministry of Education and Sports, 2008). The effective teaching of geometry at this level is crucial, as it lays the foundation for learners' success in mathematics and related subjects in higher grades.

In the Ugandan context, geometry teaching is further complicated by the socio-economic and cultural factors that influence education. Many primary schools, particularly in rural areas, lack the necessary resources and qualified teachers to provide high-quality mathematics instruction (Lauwerier & Akkari, 2015). Additionally, the

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pedagogical approaches used in teacher training colleges often do not adequately prepare pre-service teachers for the classroom realities. For instance, in 2014, the Uganda National Examinations Board (UNEB) reported that only 51.6% and 44.2% of Senior Two students achieved the desired proficiency levels in measures and geometry, respectively (UNEB, 2014). Moreover, fewer than 10% of the students demonstrated the ability to compute the number of sides of a regular polygon or correctly identify the side of a right-angled triangle adjacent to a given angle. Komakech (2017) identified several challenges among teachers, including inadequate skills in test construction and marking, limited competencies in teaching, deficiencies in content knowledge, and the ability to interpret concepts effectively. This was evident with all sampled teachers failing the quiz on the sum of the series compared to only 1.8% of the students who passed. As a result, many teachers enter the profession with insufficient training in both content and pedagogy, leading to suboptimal teaching practices and poor learner outcomes in geometry (Ball, Thames & Phelps, 2008; Komakech & Osuu, 2014; Nilsson & Karlsson, 2019). Given these challenges, this study addresses this gap by exploring how pre-service mathematics teachers in Uganda use PCK to teach geometry in primary six classrooms.

Statement of the Problem

Teaching geometry in primary education poses considerable challenges, especially in cases where pre-service teachers aren't adequately prepared. Geometry is a crucial part of the mathematics curriculum, playing a vital role in developing students' spatial reasoning and problem-solving abilities, which are essential for success in higher education and STEM-related careers (Komakech & Osuu, 2014; Jones & Tzekaki, 2016). Unfortunately, the effectiveness of geometry instruction is often compromised by pre-service teachers' limited Pedagogical Content Knowledge (PCK). Shulman (1986) emphasized the significance of PCK in helping educators translate subject matter knowledge into accessible forms for learners.

In Uganda, many pre-service teachers struggle to acquire and apply PCK effectively, hampering their ability to teach geometry concepts successfully in primary classrooms (Akyeampong et al., 2003; Komakech & Osuu, 2014; UNEB, 2014). The deficiencies in PCK are particularly evident in pre-service teachers' challenges in addressing common misconceptions, using appropriate examples and analogies, and delivering clear explanations in geometry lessons (Hill, Ball, & Schilling, 2008; Walshaw, 2012). Furthermore, Uganda's Initial Teacher Education (ITE) programs prioritize subject matter over pedagogical content knowledge, leaving teachers ill-prepared to meet classroom demands (Akyeampong et al., 2003; Komakech, 2017). This gap between theoretical knowledge and practical teaching skills is especially prominent in primary six geometry instruction, where pre-service teachers often struggle to translate their mathematical understanding into effective teaching practices (Nilsson & Karlsson, 2019; Marikyan, 2023). Therefore, this study aims to explore how pre-service mathematics teachers in Uganda utilize PCK to teach geometry to primary six learners, to identify areas for improvement in teacher education programs, and to provide recommendations for enhancing the quality of mathematics instruction in primary schools.

Research Objectives

The primary objective of this study is to explore how pre-service mathematics teachers use Pedagogical Content Knowledge (PCK) to teach geometry to six primary learners. The specific objectives of the study are:

- To explore the use of examples, explanations, demonstrations, illustrations, and analogies by pre-service mathematics teachers in teaching geometry.
- To find out learners' misconceptions in geometry and know how pre-service mathematics teachers address learners' misconceptions in geometry.

By addressing these objectives, the study aims to contribute to the broader body of knowledge on teacher education and provide practical recommendations for enhancing the training of pre-service mathematics teachers in Uganda.

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METHODOLOGY

The study employed a qualitative research approach, particularly suited to exploring complex phenomena in their natural settings (Creswell, 2013). This approach was chosen because it allows for a deep, contextual understanding of the variables in question, including pre-service mathematics teachers, their Pedagogical Content Knowledge (PCK), and their geometry teaching. A qualitative approach is invaluable when focusing on processes, meanings, and the participants' perspectives, rather than quantifiable outcomes (Dul & Hak, 2008; Cohen, Manion, & Morrison, 2011; Yin, 2009). Specifically, the study adopted a case study design, a standard method in educational research that enables an in-depth exploration of the subject matter within its real-life context (Yin, 2009). The case study was conducted in a Primary Teachers' College (PTC) and an associated primary school in Asuret Sub County, Soroti District, Eastern Uganda. The choice of a case study design was motivated by the desire to understand how pre-service teachers apply PCK in teaching geometry, allowing for a detailed examination of their instructional practices and challenges.

Selection of Participants

The selection of participants was purposeful, targeting those most likely to provide rich, relevant data for the study. The primary participants were four pre-service mathematics teachers, two males and two females, enrolled in a Grade Three Teachers' Certificate course. These participants were chosen because they had undergone some pedagogical training and were in their second year of study, typically when learners begin to apply pedagogical theories in practical teaching contexts. The inclusion of female pre-service teachers was particularly emphasized to encourage gender balance in teaching and to explore whether female teachers might influence girls' participation in geometry lessons, a subject often perceived as challenging by female learners (Akyeampong et al., 2003; Komakech & Osuu, 2014). Additionally, the study involved 68 learners from the primary school where the pre-service teachers conducted their teaching practice. The learners, aged between 10 and 14 years, were divided into two streams, ensuring that all learners could benefit from the four planned geometry lessons. This particular school was selected based on its proximity to the college and the researcher's professional connection with both institutions, facilitating easier access and a more collaborative research environment.

Data Collection Methods

This study utilized multiple qualitative data collection methods to enhance the credibility and reliability of its findings, in line with the principles of triangulation in qualitative case studies (Denzin & Lincoln, 2018; Komakech et al., 2024). Focusing on the use of Pedagogical Content Knowledge (PCK) by pre-service mathematics teachers in geometry instruction, data were gathered through observations, interviews, and document analysis. A systematically developed and validated observation checklist was created to ensure reliability and alignment with the study's objectives (Hammersley, 2006). This checklist aimed to capture real-time interactions between teachers and learners, the instructional strategies employed, student engagement levels, and how teachers addressed misconceptions (Cohen, Manion, & Morrison, 2002). Each item on the checklist was carefully aligned with the research questions, ensuring a structured and focused approach. To enhance validity, subject matter experts reviewed the checklist for clarity and relevance, and a pilot study was conducted in a different primary school to refine any ambiguous items. Inter-rater agreement was employed to ensure consistency in observations, making the checklist a robust tool for documenting qualitative data on the application of PCK in primary six geometry classrooms (Gay, Mills, & Airasian, 2011; Shulman, 1986; Fraenkel, Wallen, & Hyun, 2019).

Following classroom observations, semi-structured interviews were conducted with pre-service teachers to gain deeper insights into their teaching experiences and perceptions of pedagogical content knowledge (PCK) in geometry instruction. These interviews offered a flexible yet structured approach to exploring how teachers addressed misconceptions, represented geometric concepts, and implemented instructional strategies (Yin, 2009; Komakech & Osuu, 2014). The interview questions

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were thoughtfully crafted to delve into key aspects of geometry teaching, enabling participants to elaborate on their instructional decisions and classroom challenges. To augment the observational and interview data, a document analysis was performed, focusing on the Primary Six curriculum, mathematics syllabi, teacher guides, and student textbooks. This analysis aimed to evaluate how these instructional resources either supported or hindered the effective use of PCK in geometry teaching. A document analysis protocol was employed to assess the relevance, currency, and quality of these materials, ensuring they aligned with PCK principles (Bowen, 2009; Komakech et al., 2024). The integration of these three data collection methods provided a comprehensive understanding of how pre-service teachers applied PCK in their teaching, resulting in a well-rounded analysis of their instructional practices and challenges.

Data Analysis

The data collected through observations, interviews, and document analysis were subjected to thematic analysis, a method well-suited for identifying, analyzing, and reporting patterns within qualitative data (Braun & Clarke, 2006). The analysis process began with the transcription of interview data and the organization of observation notes and documents. The data were then coded, with codes derived from both the research questions and the data itself, allowing for a mix of deductive and inductive analysis. According to Braun & Clarke (2006), thematic analysis involves several steps:

- *Familiarization with Data:* The researchers immersed themselves in the data by reading and re-reading transcripts, observation notes, and documents to identify significant patterns and themes.
- *Generating Initial Codes:* Initial codes were generated based on recurring topics related to PCK, such as using examples, handling misconceptions, and the effectiveness of instructional strategies.
- *Searching for Themes:* The codes were then collated into broader themes that captured the essence of the data. For example, themes such as "Challenges in Teaching Geometry" and "Strategies for Addressing Misconceptions" emerged from the analysis.
- *Reviewing Themes:* The identified themes were reviewed to ensure they accurately reflected the data and were relevant to the research questions. This step also involved refining the themes and sub-themes to capture the nuances in the data better.
- *Defining and Naming Themes:* The themes were clearly defined and named, and the findings were organized around these themes to address the research questions.

The thematic analysis results were used to explore how pre-service mathematics teachers utilized PCK in their geometry lessons, how they addressed learner misconceptions, and the effectiveness of their instructional strategies. The analysis also provided insights into the challenges faced by these teachers and the potential areas for improvement in their training.

Ethical Considerations

This study adhered to rigorous ethical standards to ensure the research process's integrity and participants' protection. Before data collection, ethical clearance was obtained from the Aga Khan University Research Ethics Committee (REC). Subsequently, permissions were sought from the Uganda National Council for Science and Technology (UNCST) and the participating educational institutions. The principles of respect for persons and justice were ensured through the study. Informed consent was obtained from all participants, including the pre-service teachers, school administrators, and learners, ensuring they were fully aware of the study's purpose, procedures, and right to withdraw without penalty. The study also maintained confidentiality and anonymity by using pseudonyms for all participants and ensuring that personal identifiers were not included in the data or reports. Additionally, the researcher upheld the principles of honesty and transparency throughout the study, including in the data collection, analysis, and reporting processes. Reciprocity was also observed by providing professional development

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workshops to the schools involved, thereby contributing to the educational community in which the research was conducted. Therefore, this compliance enhanced the study's legitimacy and reinforced the trust of stakeholders, including academic institutions and policymakers, in the findings. Additionally, the commitment to confidentiality and anonymity, achieved through pseudonyms and the exclusion of personal identifiers, protected participants' privacy and encouraged openness in responses, ultimately improving the reliability of the qualitative data.

PRESENTATION OF FINDINGS

The study presents findings on respondents' characteristics, study area, and how pre-service mathematics teachers apply Pedagogical Content Knowledge (PCK) to teach geometry. It focuses on their strategies for addressing and correcting learners' misconceptions through real-time interventions. The analysis delves into how PCK was used to enhance geometry instruction and the effectiveness of on-the-spot teaching strategies in mitigating common misunderstandings among primary six learners.

Respondents' Characteristics and Study Area

The study's findings were drawn from qualitative data gathered from four (4) pre-service mathematics teachers, each referred to by pseudonyms (Jojo, Ngo, Bit, and Gil) to protect their anonymity. These participants were selected from a primary school in the Soroti district of Eastern Uganda, with a specific focus on their methods and experiences in teaching geometry to six primary school students. The teachers employed different approaches, providing a range of perspectives on using PCK in a natural classroom setting. The study examined how these pre-service teachers utilized examples, explanations, illustrations, analogies, and demonstrations in their teaching and how they tackled common misconceptions in geometry. The participants' diverse experiences and teaching strategies offered a valuable dataset, which was analyzed to comprehend the effectiveness and challenges of implementing PCK in the context of Ugandan primary education.

Pedagogical Strategies for Teaching Geometry

This section explores how pre-service mathematics teachers applied PCK to effectively teach geometry, emphasizing the five (5) pedagogical strategies to enhance learner's understanding and engagement.

Use of Examples in Geometry

Using examples in teaching geometry is a critical pedagogical strategy that aids in bridging abstract concepts with tangible understanding. As evidenced by the qualitative data collected through interviews and observation, examples were used to define and illustrate abstract ideas, making them more tangible and understandable for students in this study; examples serve as fundamental tools for defining and reinforcing geometric concepts. For instance, participants like Teacher Jojo and Teacher Ngo utilized examples to make geometry more relatable and understandable for their students. Jojo defined a line as *"a path of points that continue indefinitely in the opposite direction,"* while Ngo described parallel lines as *"lines that do not intersect or meet."* These definitions were not merely theoretical; they were coupled with practical illustrations, such as using pieces of grass to represent parallel lines, underscoring the importance of connecting classroom learning to real-life situations.

However, while examples are powerful tools in teaching geometry, the study also reveals potential pitfalls. Not all examples are equally effective in conveying basic geometric ideas. The choice of examples must be carefully graded and selected to ensure they are appropriate for the concept being taught. Poorly chosen examples can lead to misconceptions or inadequate development of geometric understanding, highlighting the need for teachers to develop a keen sense of which examples will most effectively prompt reasoning and conjecturing among students. Furthermore, it's important to note that using examples in teaching geometry goes beyond simple definitions. As highlighted by document analyses from the study, examples in lesson plans and curricula are strategically employed to suggest learning activities that foster deeper engagement with geometric concepts. This approach aligns with contemporary educational practices emphasizing the need for

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concrete, visual, and relatable examples to enhance learners' comprehension and retention of abstract ideas. Finally, strategically using examples in geometry instruction is vital for developing students' understanding of mathematical concepts. Teachers must be deliberate in their choice of examples, ensuring they are aligned with the learning objectives and appropriate for the student's level of comprehension. As demonstrated by this study, when used effectively, examples can significantly enhance the teaching and learning of geometry, making abstract concepts more concrete and understandable for students.

Use of Illustrations in Geometry

Illustrations are crucial in teaching geometry, providing visual representations that make abstract concepts more tangible and understandable for learners. The qualitative results from this study highlight the diverse ways pre-service teachers employ illustrations to enhance geometry learning. Using diagrams, concrete objects, and environmental elements as teaching tools underscores the importance of connecting geometric concepts to the physical world, facilitating deeper comprehension among learners. For instance, the study reveals that pre-service teachers like Jojo and Ngo effectively utilized illustrations to clarify geometric ideas. Jojo's use of grass to illustrate lines and Ngo's employment of metallic bars and grass to demonstrate parallel lines exemplify how tangible, real-world objects can bridge the gap between abstract geometric concepts and learners' understanding. Therefore, by using objects familiar to learners, teachers help make geometry's abstract nature more relatable and accessible.

Furthermore, the study shows that illustrations are not limited to simple diagrams but can extend to more complex and interactive methods. For example, Teacher Bit's use of a paper box to demonstrate skew lines and the classroom walls to differentiate between parallel and skew lines illustrates how teachers can use the environment and physical models to correct misconceptions and solidify learners' understanding. Using the classroom as a learning tool is particularly effective in addressing common misunderstandings in geometry, as it allows learners to see the practical application of geometric principles in their surroundings. The study's document analysis further supports the importance of illustrations in teaching geometry. Including illustrative content in teachers' schemes of work suggests that illustrations are planned and integral parts of the instructional process. This planning ensures that illustrations are spontaneous or ad hoc, carefully selected, and designed to support specific learning objectives. As the findings indicate, these illustrations guide learners through complex geometric concepts, aiding in developing spatial reasoning and conceptual understanding.

Finally, the study confirms the effectiveness of using illustrations in geometry instruction. For example, the variety in the types of illustrations used, ranging from diagrams to physical objects, demonstrates the flexibility and creativity of the teachers but also suggests that a more structured framework for using illustrations could enhance their effectiveness. Thus, teachers must be equipped with the skills to select the most appropriate illustrations for different concepts, ensuring that these tools effectively address learners' learning needs and misconceptions.

Use of Explanations in Geometry

The findings of this study highlight the critical role of explanations in the teaching of geometry to primary six learners. Explanations are an essential pedagogical tool that serves multiple functions in the classroom, including clarifying concepts, guiding instructional procedures, and enhancing learners' reasoning and problem-solving skills. The evidence from observations, interviews, and document analysis suggests that pre-service teachers effectively utilize explanations to facilitate learners' understanding of geometric concepts.

One key finding is that teachers use explanations to make abstract geometric concepts more accessible to learners. For instance, Teacher Ngo emphasized the importance of explanations in helping learners grasp the content of geometry lessons and in guiding them through specific teaching procedures. Similarly, the teachers' use of explanations to articulate and reinforce the correct procedures for measuring lines, as observed in Jojo's lesson, further illustrates the practical application of this pedagogical strategy in

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enhancing learners' mathematical accuracy and confidence.

The study also reveals that explanations are crucial in assessing and correcting learners' understanding. Teacher Bit's use of explanations to correct the spelling of geometric vocabulary and to frame and refocus questions during a lesson on skew lines highlights the dynamic nature of explanations in the classroom. This approach clarifies learners' misconceptions and fosters an interactive learning environment where learners are encouraged to engage critically with the content. The document analysis further indicates that explanations are integral to the instructional strategies outlined in the primary six curriculum and syllabi. The curriculum documents for primary six analyzed in this study demonstrate that explanations are explicitly recommended for teaching and assessing geometric concepts. However, the study also identifies a potential gap in the curriculum, particularly the limited time allocated for teaching geometry, which may constrain the effectiveness of explanations in fully addressing the learning needs of learners. This observation suggests that while explanations are a powerful tool, their impact may be limited by structural constraints within the educational system.

The findings also highlight the importance of explanations in developing learners' procedural knowledge and conceptual understanding. The study observed that explanations are used to verify geometric proofs, underscores their role in fostering learners' ability to reason logically and apply mathematical principles to solve problems. This is particularly important in the context of geometry, where learners must not only understand the properties of shapes and figures but also be able to apply these concepts in practical situations. The study concludes that the use of explanations by pre-service teachers in teaching geometry to primary six learners is a critical component of effective pedagogy. This is because explanations clarify complex concepts, guide instructional practices, and correct misconceptions, enhancing learners' overall understanding and performance in geometry.

Use of Demonstrations in Geometry

The findings from this study underscore the critical role of demonstrations in teaching geometry to primary six learners. As employed by pre-service teachers, demonstrations serve as a powerful instructional strategy that bridges the gap between abstract geometric concepts and practical understanding. The qualitative data reveal that pre-service teachers frequently use demonstrations to help learners grasp the content more effectively. For instance, both teachers (Jojo, Ngo, Bit, and Gil) emphasized the importance of demonstrating the correct use of tools like rulers to measure lines accurately. Furthermore, Teacher Gil and Bit's focus on showing learners how to handle a pair of compasses, for example, correctly, highlights the pedagogical value of demonstrations in ensuring that learners develop the necessary skills to engage with geometry meaningfully.

In addition to the hands-on aspects of geometry instruction, the document analysis provides evidence that pre-service teachers also use demonstrations to organize and sequence their teaching content. By breaking down complex topics into simpler, more manageable components, teachers can scaffold learning to promote deeper understanding and retention of geometric concepts. Researchers also observed that demonstrations extend beyond classroom instruction to include the assessment of learner work. For instance, pre-service teachers were observed using demonstrations as part of their assessment practices, such as marking learners' work and providing feedback on their understanding of geometric principles. This integration of demonstrations into teaching and assessment highlights their versatility and effectiveness as a pedagogical tool in geometry education.

Finally, it's important to note that demonstrations play a pivotal role in teaching geometry's preparation and planning stages. Demonstrations also help to clarify abstract concepts, model the correct use of mathematical tools, and provide a structured approach to content delivery Drawing from observation and interview, pre-service teachers utilize demonstrations to sequence topics logically and to plan lessons that progressively build on learners' prior knowledge. This approach ensures

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that learners are not overwhelmed by the complexity of geometric concepts and can develop their understanding incrementally, a fundamental principle of effective mathematics instruction.

Use of Analogies in Geometry

Using analogies in teaching geometry to primary six learners represents a powerful pedagogical strategy that helps bridge the gap between abstract concepts and learners' everyday experiences. The findings from the study reveal that pre-service mathematics teachers effectively employed analogies to clarify and reinforce geometric ideas. For instance, Teacher Ngo's comparison of parallel lines to iron bars in a window and pieces of grass is an excellent example of how analogies can make abstract concepts more tangible and relatable for learners.

Analogies were also used to address more complex geometric concepts, such as skewed lines. Teacher Bit's demonstration, which involved using an empty box and pieces of grass to illustrate the idea of skew lines, further exemplifies how analogies can simplify difficult/complex concepts. Teachers can demystify challenging ideas by relating geometric figures to familiar objects and making them accessible to young learners. Hence, the interactive nature of the analogies used, such as having learners physically walk side by side to represent parallel lines, encourages active participation and reinforces learning through kinesthetic experiences.

This study also highlights the necessity for careful selection and clear explanation when employing analogies. Misunderstandings can arise if analogies are not well chosen or the connections between the analogy and the geometric concept are unclear. For example, the potential confusion between parallel and skewed lines could lead to misconceptions if not adequately addressed. In conclusion, analogies facilitate deeper cognitive processing by making new information more relatable and easier to grasp. However, to maximize their effectiveness, analogies must be carefully selected and clearly explained to ensure they reinforce accurate conceptual understanding.

Learners' Misconceptions in Geometry and On-spot Pedagogical Strategies

Misconceptions in geometry, often characterized as inaccurate understandings or interpretations of geometric concepts, are common among primary students. These misconceptions can significantly hinder the learning process and impede the development of a solid mathematical foundation. This study highlights several common misconceptions recorded among primary six learners and the strategies employed by prospective math teachers to address them. One prevalent misconception was the improper use of rulers during measurement tasks. Learners frequently misused the ruler, resulting in inaccurate measurements. Teacher Jojo tackled this by providing explicit instructions on proper ruler placement, emphasizing the alignment of the zero mark with the starting point of the line being measured. The second misconception was the common mistake involved learners counting the divisions on the ruler rather than the spaces between them when measuring lengths. Jojo demonstrated the correct method to address this, guiding students to focus on counting the spaces that represent the actual measurement indicators instead of the divisions.

Thirdly, researchers observed the third misconception: learners face difficulties distinguishing between parallel and skewed lines, often mistaking one for the other. Teacher Bit effectively tackled this misunderstanding by using real-life examples, such as classroom walls and pieces of grass, to visually illustrate the disparities between parallel and skewed lines. Connecting abstract geometric concepts to tangible real-world objects helps learners form more accurate mental models, which is crucial for comprehension and retention. Finally, group work proved to be a valuable approach to addressing these misconceptions. Teachers encouraged learners to collaborate and facilitated peer-assisted learning, allowing learners to discuss and compare their understanding of geometric concepts. Therefore, the findings imply that pre-service mathematics teachers possess the necessary pedagogical content knowledge to address common misconceptions in geometry effectively. Through direct instruction,

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real-life examples, and collaborative learning strategies, these teachers showcased their ability to rectify learners' misunderstandings and promote a deeper comprehension of geometric concepts.

DISCUSSION OF FINDINGS

The results of this study demonstrate that pre-service teachers effectively utilize examples to connect abstract geometry concepts with real-life applications. Hill, Ball & Schilling (2008) and Komakech & Osuu (2014) underline the importance of concrete examples in improving learners' engagement and comprehension of mathematical concepts. However, the findings also highlight a limitation in the range of examples used by pre-service teachers, indicating the need for further development. Moreover, recent studies by Buchbinder, McCrone, Capozzoli & Butler (2021) and Sinclair, Watson, Zazkis & Mason (2011) support this perspective by advocating for a broader range of examples to meet diverse learning needs, essential for promoting a deeper conceptual understanding among students. These findings suggest that while pre-service teachers are progressing in connecting geometry to real-world contexts, there is a critical need for more targeted training to expand their example repertoire, enhancing their capacity to address diverse learner needs and deepen students' conceptual understanding of geometric principles.

Pre-service teachers effectively utilized illustrations to elucidate geometric concepts, primarily through models and diagrams. This method aligns with literature emphasizing the role of visual aids in enhancing learners' spatial reasoning and understanding of geometric properties (Clements & Battista, 1992; Weber & Larsen, 2008; Clements & Sarama, 2011; Coles & Sinclair, 2019; Geçici & Türnüklü, 2021; Marikyan, 2023). According to Clements & Sarama (2011), visuals play a crucial role in fostering spatial reasoning skills essential for mathematical problem-solving. Diagrams, graphs, and models help students visualize mathematical relationships and concepts, enhancing their spatial reasoning abilities. Furthermore, using real-life examples, such as classroom walls and pieces of grass, to visually distinguish between these concepts is in line with Piaget's (1970) theory, which posits that connecting abstract concepts to tangible objects enhances comprehension and retention. Nonetheless, the study noted that illustrations were employed less frequently than examples. Alcock & Simpson (2001) suggest that this underutilization may stem from teachers' lack of confidence or experience in utilizing visual tools proficiently. Similarly, findings from the study on pre-service teachers in Vietnam reflect a similar trend where limited training and experience lead to the underutilization of visual tools (Geçici & Türnüklü, 2021; Carter et al., 2023). These findings are consistent with earlier research by Komakech and Osuu (2014), highlighting the inadequacy of instructional materials and their impact on the teaching and learning of science and mathematics in Uganda.

Explanations have emerged as a primary pedagogical tool, with pre-service teachers utilizing them to reinforce learning, clarify concepts, and lead learners through reasoning processes. This is consistent with existing research highlighting the importance of clear, well-structured explanations in enhancing student comprehension and involvement in mathematics (Hill, Ball, & Schilling, 2008; Skemp, 2012; Komakech & Osuu, 2014; Marikyan, 2023). Scholars such as Ball, et al (2008) also support using explanations to foster higher-order thinking and reflective learning in mathematics education. To further improve the effectiveness of explanations in mathematics education, it is crucial for educators not only to emphasize clarity and structure but also to provide opportunities for students to actively engage with and apply these explanations, thus promoting deeper understanding and long-term retention of mathematical concepts.

Analogies were utilized infrequently, suggesting a gap in the teachers' pedagogical content knowledge. Although analogies are powerful tools for connecting new information to student's prior knowledge, their underutilization indicates a need for more targeted training on effectively integrating analogies into geometry instruction. This approach is consistent with established educational theories. For example, Richland & McDonough (2010) propose that analogies support learning by linking new information to existing knowledge, thus

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improving comprehension and retention. Gray & Holyoak (2021) conducted a study identifying analogy as a potent tool for promoting conceptual understanding and transfer in STEM. They posited that well-crafted analogical comparisons direct attention to the causal-relational structure of STEM concepts and facilitate the ability to make inferences based on a familiar source domain that can be applied to a new target domain. The authors also emphasized the importance of using analogy thoughtfully, considering students' prior knowledge and cognitive resources. Contributing to the same discourse, studies suggest that analogies can significantly enhance conceptual understanding when used appropriately, but they require careful selection and explanation to avoid misconceptions (Gray & Holyoak, 2021; Sampson & Blanchard, 2012). As such, teachers need specialized professional development in strategically using analogies, ensuring that they are thoughtfully chosen and effectively explained to optimize student understanding while minimizing the risk of reinforcing misconceptions.

The study highlights several misconceptions in geometry that are common among primary six students, such as inverting the ruler when measuring lengths and confusing parallel lines with skew lines. According to Hiebert & Grouws (2007), these misunderstandings reflect deep-seated challenges in grasping fundamental geometric concepts, which are crucial for more advanced mathematical learning. These challenges were identified in early mathematics education four decades ago. Posner, Strike, Hewson & Gertzog (1982) reported that students often misinterpret the divisions on a ruler, mistakenly counting the divisions rather than the spaces between them, a common issue. Similarly, the conflation of parallel and skew lines indicates a lack of clarity in understanding geometric relationships, a problem that has been well-documented in geometry instruction (Shulman, 1986; Komakech, 2017; Sinclair, et al. 2017; Nilsson & Karlsson, 2019). These findings underscore the prevalence of misconceptions in early geometry education and the potential barriers they create for students' overall mathematical development (Posner et al. 1982; Sinclair et al., 2017). However, Ball et al. (2008) noted that addressing these misconceptions is essential to ensure students build a solid foundation in geometry, which is vital for future success in more complex mathematical areas. This view was supported by researchers (such as Pinto & Pacheco, 2015; Hoth, Larrain & Kaiser, 2022; Sarama & Clements, 2022), who emphasized the importance of recognizing and correcting these errors early in the learning process to prevent them from becoming entrenched misconceptions that could hinder long-term understanding.

The study emphasizes the significant challenges and strategies involved in addressing learners' misconceptions in geometry, particularly in the context of primary six education. For instance, the misconception of misaligned ruler usage during measurement tasks leads to inaccurate measurements. This resonates with the existing literature on common errors in mathematical tasks, indicating the need for direct instructional intervention (Hiebert & Grouws, 2007). The effectiveness of group work as a strategy to address these misconceptions is also noteworthy. Johnson & Johnson (2009) revealed that collaborative learning environments enable peer-assisted learning, fostering a more profound and more collaborative learning experience. The findings indicate that pre-service teachers, through direct instruction, real-life examples, and cooperative learning strategies, can effectively correct misconceptions in geometry. The findings contrast with Darling-Hammond (2010), who observed that pre-service teachers often struggle with demonstrations due to insufficient rehearsal, highlighting the necessity for teacher training programs to provide more opportunities for practice and feedback—an approach supported by research on adequate teacher preparation. However, Ball et al. (2008) reported that these strategies are critical in ensuring that learners develop a solid and accurate understanding of geometric concepts, which is foundational for their future success in mathematics and related fields.

CONCLUSION

This study provides valuable insights into pre-service mathematics teachers' application of Pedagogical Content Knowledge (PCK) in teaching geometry to primary six learners in Uganda. The findings highlight that while pre-service teachers are

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beginning to effectively use examples, illustrations, explanations, and demonstrations to facilitate learner understanding of geometric concepts, there are notable gaps, particularly in the use of analogies and in addressing learner misconceptions. These gaps suggest a need for more targeted pedagogical training and support, particularly in developing a comprehensive understanding of how to integrate analogies into instruction and effectively identify and correct misconceptions in real-time classroom settings.

The study underscores the critical role of thorough preparation and rehearsal in successfully delivering instructional strategies, particularly demonstrations, essential for making abstract concepts tangible for young learners. Additionally, the findings highlight the importance of expanding the repertoire of examples and illustrations used in instruction to cater to diverse learning needs and foster deeper conceptual understanding. Finally, the study contributes to the existing literature on PCK by offering specific recommendations for improving teacher training programs, particularly in the context of developing countries where resources and support may be limited. It calls for a more focused approach to teacher education that emphasizes the development of robust PCK, including the ability to use analogies and address misconceptions effectively.

Recommendations and Areas for Future Research

Based on the study's findings, several recommendations and future research areas were made, including: Firstly, the teacher education programs should emphasize the development of PCK, particularly in using analogies and addressing misconceptions. This could be achieved through increased opportunities for practice teaching, feedback sessions, and reflection. Training programs should also encourage pre-service teachers to rehearse their demonstrations before classroom implementation to build confidence and reduce errors during lessons. Secondly, the curriculum for teacher training should include specific modules on using different PCK components in teaching geometry. This could include case studies, practical examples, and workshops on effectively using examples, illustrations, explanations, and demonstrations. Additionally, there should be a focus on integrating real-life applications of geometry into the curriculum to help pre-service teachers relate abstract concepts to everyday situations. Finally, future research should explore using PCK in different educational contexts to better understand how cultural and regional differences impact geometry teaching. Additionally, research could investigate the long-term impact of enhanced PCK training on pre-service teachers' effectiveness in the classroom. Another area for future research could be exploring how digital tools and resources can support the development and use of PCK in teaching geometry, particularly in resource-constrained settings. Finally, future research should identify best practices for supporting pre-service teachers in their professional development and enhancing their capacity to teach complex mathematical concepts effectively.

Limitations of the Study

One of the primary limitations of this study is the small sample size, which consisted of only four pre-service teachers and one primary school. While this allowed for a detailed, in-depth exploration of the teachers' use of PCK, it limits the generalizability of the findings. Future research with a larger sample size across multiple schools would help validate these findings and provide a more comprehensive understanding of how pre-service teachers use PCK in geometry instruction. Another limitation is the specific educational context in which the study was conducted. The study was conducted in a rural area of Uganda, which may have unique educational challenges and resources that differ from other settings. This context-specific nature of the study means that the findings may not be directly applicable to other regions or countries with different educational systems and cultural contexts. Future studies could explore similar research in diverse settings to compare how PCK is used in different educational environments.

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