



Original Article

Heat Exchanger Network Synthesis Using Node-based Non-Structural Model With Enhanced Dynamics for Stream Matching

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Node-based nonstructural models (NNMs) for heat exchanger network (HEN) synthesis realize flexibilities for stream matching across the entire ranges of process streams, as the concept of stages is not used. These models can also accelerate the efficiency of optimization algorithms. However, since nodes are considered to be in fixed positions, heat exchangers tend to crowd in the feed regions of the process streams during the later stages of optimization. This crowding hinders the generation of new heat exchangers in those areas, eventually interrupting the randomness of the NNM and impeding structural optimization. This paper proposes a mechanism for adding split groups within existing node groups in process streams to allow for freer generation of new stream matches and reduce exchanger clustering. The random walk algorithm with compulsive evolution (RWCE) is utilized for HEN optimization. The algorithm is particularly suitable because it can evolve only existing heat exchangers while also generating new ones independently. Examples from the literature are solved to illustrate the applicability of the proposed modifications to NNM and the results compare well with solutions reported in the literature.

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INTRODUCTION

Energy demands for operating chemical processing plants are a great concern as they count the largest portion of energy consumption in the plants (Mtebwa and Ichwekeleza, 2022). Utility energy requirements may need the use of oil as a fuel whose crisis scenario is known all over the world currently. Heat exchanger networks (HENs) are essential in industrial process designs to maximize the process-to-process heat recoveries and minimize the utility (energy) requirements. To achieve the best HENs, different approaches have been followed including the pinch method (Linnhoff and Hindmarsh, 1983), mathematical programming methods based on pinch concepts (Papoulias and Grossmann, 1983), and mathematical methods based on superstructures (Yee and Grossmann, 1990). As the superstructure methods define initial structures from which different design alternatives are selected, they can be prone to missing necessary design alternatives (Lotfi, 2010). For that matter, non-structural models for HEN synthesis have been researched. The HENs without stream splits (Chakraborty and Ghosh, 1999) and those with splits (Pariyani et al., 2006) obtained using the randomized algorithms have been reported in the literature in which cases no definite structures were predefined at the start. In their work, Pariyani et al. (2006) simplified the HENs by permitting a single two-branch split per stream per network along with isothermal mixing of stream branches. They further limited the number of process heat exchangers to some maximum in each of the streams. In the work of Toffolo (Toffolo, 2009), a method for synthesis of HENs with unconstrained topology based on graph representations was developed and tested for relatively small-scale examples from the literature.

From an optimization viewpoint, simultaneous HEN synthesis methods are usually grouped as mixed-integer non-linear programming models that are mostly solved by stochastic methods. Different stochastic optimization algorithms have been employed for HEN synthesis as reported in the literature including simulated annealing (SA)

(Peng and Cui, 2015), genetic algorithm (GA) (Ravagnani et al., 2005), particle swarm optimization (PSO) (Silva et al., 2010), differential evolution (DE) (Yerramsetty and Murty, 2008) and random walk algorithm with compulsive evolution (RWCE) (Xiao and Cui, 2017) that were used singly, as well as hybrid algorithms such as GA and SA (Luo et al., 2009), GA and PSO (Huo et al., 2013; Pavão et al., 2016), SA and PSO (Pavão et al., 2017), and GA and DE (Aguitoni et al., 2018) have been used to solve the HEN synthesis problem.

Node-based non-structural models (NNMs) (Xu et al., 2020a; Xu et al., 2020b) have been shown to be promising for the synthesis of HENs. According to Xu et al. (2020a), a single-stage NNM can address the limitations in stream matching caused by the prohibition of cross-stage matching in the stage-wise superstructure (SWS) model (Yee and Grossmann, 1990). The model (i.e. NNM) is thus, capable of achieving more freedom in stream matching, by randomly matching the pairs of nodes in streams, compared to SWS HEN models. NNM can, therefore, efficiently obtain high-quality solutions, to a large number of HEN optimization case studies. To realize sufficient solution space and reduce the number of variables for the problem, Xu et al. (Xu et al., 2020c) improved NNM into a model that allows to adjust the number of nodes by disregarding the unoccupied nodes. Xiao et al. (2020) introduced (into NNM) an additional strategy by which node locations of existing stream matches could uniformly be distributed on the process streams. The strategy aimed to realize more freedom of generation of new stream matches and expand the solution domain, but did not consider stream splits. In the later work, Xiao et al. (2021) attempted to solve the HEN synthesis problem using a node-dynamic adaptive non-structural model aiming to reach higher efficiency in optimization under a manageable solution space. Their work did not allow the splitting of process streams though. However, in the actual HEN optimization process based on NNM, heat exchange units gather near the stream sources (Xiao et al., 2021) as the model exhibits nodes

placed in fixed positions and restricts a node to be matched only once.

This study proposes modifications to NNM (Xu et al, 2020a) with stream splits to do away with heat exchanger crowding and improve stream matching freedom and network flexibility in solving the HEN synthesis problem. The study proposes a dynamic split group NNM (DSG-NNM) model to incorporate extra split groups between node groups in individual process streams thus permitting the generation of new stream matches while avoiding exchanger clustering. RWCE is applied to optimize HENs taking advantage of its capability to allow evolution of only existing heat exchangers and independently generate new heat exchangers. This optimization method has demonstrated good performance in HEN synthesis (Zhongkai et al., 2018; Kayange et al., 2020, 2021), by achieving better total annual cost (TAC) results than many other stochastic methods reported in the literature.

MATERIALS AND METHODS

The work reported in this paper involved the use of RWCE to optimize HENs based on modified NNM. The code was implemented in Fortran (CVF 6.6). This section first introduces the HEN synthesis problem, then presents the formulation of NNM and the associated modifications and further outlines the objective function and the constraints accorded.

HEN Synthesis Problem Definition

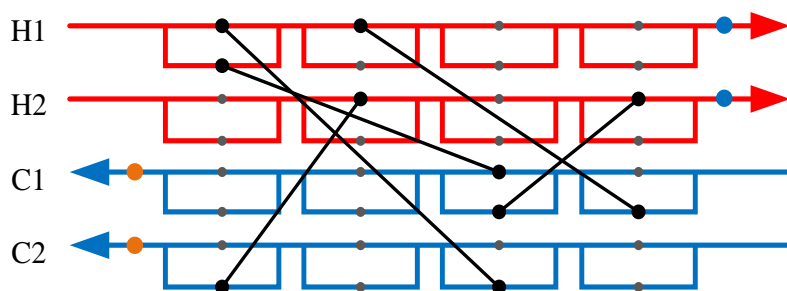
In the HEN synthesis problem, usually the following are given: sets of ‘cold’ and ‘hot’ process streams (i.e. that require heating and

cooling, respectively) with known supply and target temperatures and heat capacity flow rates; cold and hot utilities along with their respective temperature target values; the heat transfer coefficients of individual process and utility streams or overall heat transfer coefficients for each possible process heat exchanger, and the economic data including utility charges and area cost correlations for each heat exchanger. The HEN synthesis problem then pertains to determining the topology of HEN that, in the course of bringing all stream temperatures to their target values, best recovers heat energy by matching the given hot and cold process streams in order that the TAC, consisting of the utility cost and the annualized investment cost, is minimized. Stream splits are permissible and may appear in the potential HEN configurations but the costs associated with pipework are considered negligible in this paper.

The Mathematical Formulation of NNM

The schematic diagram of NNM is shown in Figure 1. The figure shows two cold streams and two hot streams, represented by blue and red line segments, respectively. An empty node (i.e. a node that is not connected to another to define a heat exchanger) is represented by a grey dot and a black line connecting the two black dots represents a heat exchanger (i.e. a hot-cold process stream match). The blue and orange dots at the end of the process fluids represent cold and hot utility exchangers, respectively. Split groups are arranged in series on a stream, and split branches within the split group are arranged in parallel. In Figure 1, both hot and cold streams have split groups with 2-way splits.

Figure 1: Schematic Diagram of NNM with Stream Splits for an F-stream Problem



The variables of the model are represented as follows. The numbers of split groups in hot and cold streams are denoted by Nd_H and Nd_C , respectively and individual groups are respectively, indexed as $nd_H = 1, 2, \dots, Nd_H$ and $nd_C = 1, 2, \dots, Nd_C$. Substreams in a split group in a hot stream are indexed as nf_H such that $nf_H = 1, 2, \dots, Nf_H$ and on cold streams as nf_C so that $nf_C = 1, 2, \dots, Nf_C$, where Nf_H and Nf_C are total numbers of split branches on hot and cold streams, respectively. For a particular hot or cold stream, if

$$Ne_i = Nf_H \times Nd_H, \quad ne_i = 1, 2, \dots, Ne_i, i \in N_H \quad (1)$$

$$Ne_{i'} = Nf_C \times Nd_C, \quad ne_{i'} = 1, 2, \dots, Ne_{i'}, i' \in N_C \quad (2)$$

$$Nt_H = Ne_i \times N_H \quad (3)$$

$$Nt_C = Ne_{i'} \times N_C \quad (4)$$

Objective Function

The optimization of HENs involves the minimization of TAC subject to the constraints

$Nf_H = 1$ or $Nf_C = 1$, then the stream exhibits no splits and, in such a case, the number of split groups equals the number of nodes. Ne_i and $Ne_{i'}$ denote the numbers of nodes on a hot stream i and cold stream i' , respectively. The total number of nodes in all hot and cold streams is denoted as Nt_H and Nt_C , respectively. The corresponding mathematical relationships between the aforementioned parameters are shown in Equations (1) to (4).

presented in Equations (21) - (30). The objective function, TAC which combines the fixed investment cost and operating cost is shown in Equation (5),

$$\begin{aligned} \min TAC = \min & \sum_{i=1}^{N_H} (CFc + CAc \cdot Aec_{cu,i}^\beta) \cdot z_{cu,i} \\ & + \sum_{i'=1}^{N_C} (CFh + CAh \cdot Aeh_{hu,i'}^\beta) \cdot z_{hu,i'} \\ & + \sum_{i=1}^{N_H} \sum_{nd_H}^{Nd_H} \sum_{nf_H}^{Nf_H} (CFe + CAe \cdot Aep_{i,nd_H,nf_H}^\beta) \cdot z_{i,nd_H,nf_H} + \sum_{i=1}^{N_H} CCU \cdot Q_{cu,i} \\ & + \sum_{i'=1}^{N_C} CHU \cdot Q_{hu,i'} \end{aligned} \quad (5)$$

Where, CFc , CFe , CFh are the fixed investment costs, CAc , CAe , CAh are the area cost coefficients, β is the area cost exponent, CCU and CHU are the operating cost coefficients for cold and hot utilities respectively, and z is a binary variable 1 or 0. The value of $z = 1$ indicates that a process heat exchanger, cooler or heater exists. Aep, Aec, Aeh are the respective exchanger areas and Q is the exchanger load.

The calculation for the area of a heat exchanger is done using Equations (6) to (10), where $U_{i,i'}$ is the overall heat transfer coefficient between a hot stream i and a cold stream i' . Hot stream temperatures at the inlet and outlet of a heat exchanger are denoted as TH_{i,nd_H,nf_H}^{in} and TH_{i,nd_H,nf_H}^{out} . The mapping of the 'hot node' to the 'cold node' is represented by NL and the corresponding stream temperature at the inlet and

outlet of the cold node is expressed as

$TC_{NL((i-1) \times Ne_i + (nd_H - 1) \times Nf_H + nf_H)}^{\text{in}}$ and

$TC_{NL((i-1) \times Ne_i + (nd_H - 1) \times Nf_H + nf_H)}^{\text{out}}$.

$$Aep_{i,nd_H,nf_H} = \frac{Q_{i,nd_H,nf_H}}{U_{i,i'} \cdot (\Delta T_{lm})_{i,nd_H,nf_H}}, \quad i \in N_H, i' \in N_C \quad (6)$$

$$U_{i,i'} = \frac{hH_i \cdot hC_{i'}}{hH_i + hC_{i'}}, \quad i \in N_H, i' \in N_C \quad (7)$$

$$DTL_{i,nd_H,nf_H} = TH_{i,nd_H,nf_H}^{\text{in}} - TC_{NL((i-1) \times Ne_i + (nd_H - 1) \times Nf_H + nf_H)}^{\text{out}} \quad (8)$$

$$DTR_{i,nd_H,nf_H} = TH_{i,nd_H,nf_H}^{\text{out}} - TC_{NL((i-1) \times Ne_i + (nd_H - 1) \times Nf_H + nf_H)}^{\text{in}} \quad (9)$$

$$(\Delta T_{lm})_{i,nd_H,nf_H} = \begin{cases} \frac{DTL_{i,nd_H,nf_H} - DTR_{i,nd_H,nf_H}}{\ln(DTL_{i,nd_H,nf_H}/DTR_{i,nd_H,nf_H})}, & DTL_{i,nd_H,nf_H} \neq DTR_{i,nd_H,nf_H} \\ \frac{DTL_{i,nd_H,nf_H} + DTR_{i,nd_H,nf_H}}{2}, & DTL_{i,nd_H,nf_H} = DTR_{i,nd_H,nf_H} \end{cases} \quad (10)$$

The formula for computing the areas of coolers is given in Equation (11) with the constituent variables defined in Equations (12) to (15). For $Nf_C = 1$, the stream is not split and its outlet temperature in a conceptual group is represented

by TH_{i,nd_H}^{out} . If $Nf_C > 1$, the split branch mixing temperature at each split group outlet is represented by $TH_{i,nd_H,nf_H}^{\text{out}}$.

$$Aec_{cu,i} = \frac{Q_{cu,i}}{U_{cu,i} \cdot (\Delta T_{lm})_{cu,i}}, \quad i \in N_H \quad (11)$$

$$U_{cu,i} = \frac{h_{cu} \cdot hH_i}{h_{cu} + hH_i}, \quad i \in N_H \quad (12)$$

$$DTL_{cu,i} = \begin{cases} TH_{i,nd_H}^{\text{in}} - Tcu^{\text{out}}, & Nf_H = 1 \\ \sum_{nf_H=1}^{Nf_H} (TH_{i,nd_H}^{\text{in}} \times fH_{i,nd_H,nf_H}) - Tcu^{\text{out}}, & Nf_H \neq 1 \end{cases} \quad (13)$$

$$DTR_{cu,i} = T_{H,i}^{\text{out}} - Tcu^{\text{in}} \quad (14)$$

$$(\Delta T_{lm})_{cu,i} = \begin{cases} \frac{DTL_{cu,i} - DTR_{cu,i}}{\ln(DTL_{cu,i}/DTR_{cu,i})}, & DTL_{cu,i} \neq DTR_{cu,i} \\ \frac{DTL_{cu,i} + DTR_{cu,i}}{2}, & DTL_{cu,i} = DTR_{cu,i} \end{cases} \quad (15)$$

Similarly, Equation (16) is applied to calculate the heat transfer area of the heater, where variables in the equation are shown in Equations (17) to (20).

$$A_{hu,i'} = \frac{Q_{hu,i'}}{U_{hu,i'} \cdot (\Delta T_{lm})_{hu,i'}}, \quad i' \in N_C \quad (16)$$

$$U_{hu,i'} = \frac{h_{hu} \cdot hC_{i'}}{h_{hu} + hC_{i'}}, \quad i' \in N_C \quad (17)$$

$$DTR_{hu,i'} = Th_{u,i'}^{\text{in}} - T_{C,i'}^{\text{out}} \quad (18)$$

$$DTL_{hu,i'} = \begin{cases} Th_{u,i'}^{\text{out}} - TC_{i',nd_C}^{\text{in}}, & Nf_C = 1 \\ Th_{u,i'}^{\text{out}} - \sum_{nf_C=1}^{Nf_C} (TC_{i',nd_C}^{\text{in}} \times fC_{i',nd_C,nf_C}), & Nf_C \neq 1 \end{cases} \quad (19)$$

$$(\Delta T_{lm})_{hu,i'} = \begin{cases} \frac{DTL_{hu,i'} - DTR_{hu,i'}}{\ln(DTL_{hu,i'}/DTR_{hu,i'})}, & DTL_{hu,i'} \neq DTR_{hu,i'} \\ \frac{DTL_{hu,i'} + DTR_{hu,i'}}{2}, & DTL_{hu,i'} = DTR_{hu,i'} \end{cases} \quad (20)$$

Constraints

The objective function, which involves minimizing TAC, is subject to a set of constraints related to, among other aspects, the heat and mass balances. The constraints are outlined as follows:

(i) Heat balance in process streams

Equations (21) and (22) are adopted to ensure that the quantities of heat exchanged in individual hot and cold streams, respectively balance the respective stream's overall heat loads.

$$(T_{H,i}^{\text{in}} - T_{H,i}^{\text{out}}) \cdot CpH_i = \sum_{nd_H=1}^{Nd_H} \sum_{nf_H=1}^{Nf_H} Q_{i,nd_H,nf_H} + Q_{cu,i}, \quad i \in N_H \quad (21)$$

$$(T_{C,i'}^{\text{out}} - T_{C,i'}^{\text{in}}) \cdot CpC_{i'} = \sum_{nd_C=1}^{Nd_C} \sum_{nf_C=1}^{Nf_C} Q_{i',nd_C,nf_C} + Q_{hu,i'}, \quad i' \in N_C \quad (22)$$

(ii) Heat balance in split groups

Using Equations (23) and (24), heat balances are established for each split group

to determine the unknown temperatures of the substreams flowing through the exchangers for the hot and cold streams, respectively.

$$TH_{i,(nd_H+1)}^{in} = TH_{i,nd_H}^{in} - \frac{\sum_{nf_H=1}^{nf_H} Q_{i,nd_H,nf_H}}{CF_i}, \quad i \in N_H \quad (23)$$

$$TC_{i',(nd_C-1)}^{in} = TC_{i',nd_C}^{in} - \frac{\sum_{nf_C=1}^{nf_C} Q_{i',nd_C,nf_C}}{CF_{i'}}, \quad i' \in N_C \quad (24)$$

(iii) Heat balance in each heat exchanger

To determine the unknown temperatures of streams flowing through heat exchangers, an

energy balance is performed around each exchanger. Equations (25) and (26) are used for this case.

$$Q_{i,nd_H,nf_H} = (TH_{i,nd_H,nf_H}^{in} - TH_{i,nd_H,nf_H}^{out}) \cdot (CpH_i \times fH_{i,nd_H,nf_H}), \quad i \in N_H \quad (25)$$

$$Q_{i',nd_C,nf_C} = (TC_{i',nd_C,nf_C}^{out} - TC_{i',nd_C,nf_C}^{in}) \cdot (CpC_{i'} \times fC_{i',nd_C,nf_C}), \quad i' \in N_C \quad (26)$$

(iv) Heat balance for hot and cold utility exchangers

The energy balance for heaters and coolers located at the outlet ends of the streams indicates that the inlet temperatures for the streams exchanging heat

with utility streams are essentially the mixer outlet temperatures at the mixing points of the final split group on each stream. Heat balance for coolers and heaters are given by Equations (27) and (28), respectively.

$$Q_{cu,i} = \begin{cases} \left(\sum_{nf_H=1}^{nf_H} (TH_{i,nd_H,nf_H}^{out} \times fH_{i,nd_H,nf_H}) - T_{H,i}^{out} \right) \times CpH_i, & nf_H \neq 1 \\ (TH_{i,nd_H}^{out} - T_{H,i}^{out}) \times CpH_i, & nf_H = 1 \end{cases} \quad (27)$$

$$Q_{hu,i} = \begin{cases} \left(T_{C,i}^{out} - \sum_{nf_C=1}^{nf_C} (TC_{i',nd_C,nf_C}^{out} \times fC_{i',nd_C,nf_C}) \right) \times CpC_{i'}, & nf_C \neq 1 \\ (T_{C,i}^{out} - TC_{i',nd_C}^{out}) \times CpC_{i'}, & nf_C = 1 \end{cases} \quad (28)$$

(v) Stream split fractions

Allowing the mass flow rates to be expressed in terms of heat capacity flow rates, Equations

(29) and (30) are used to realize mass balances in split groups.

$$\sum_{nf_H=1}^{Nf_H} fH_{i,nd_H,nf_H} = 1.0, \quad i \in N_H, nd_H \in Nd_H \quad (29)$$

$$\sum_{nf_C=1}^{Nf_C} fC_{i',nd_C,nf_C} = 1.0, \quad i' \in N_C, nd_C \in Nd_C \quad (30)$$

Dynamic Split Group NNM

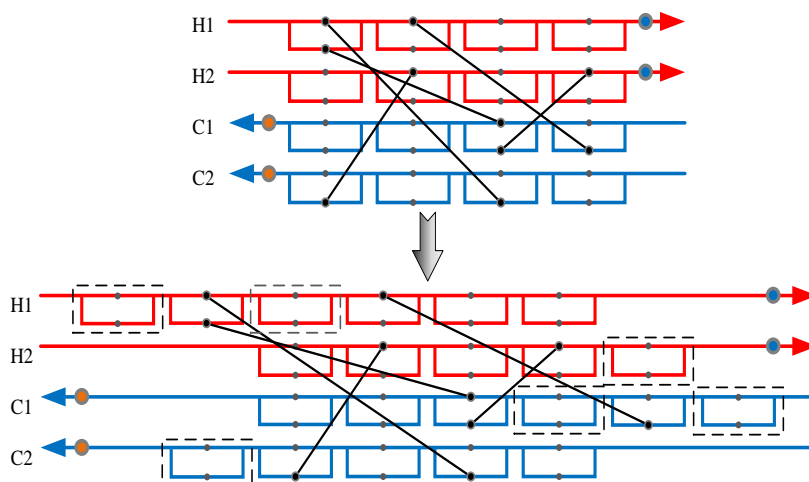
Inasmuch as NNM cannot generate new heat exchange units in some nodes, the ability of NNM to perform structural variations is insufficient. Therefore, a dynamic split group NNM (DSG-NNM) is proposed in this paper to make it possible for new process heat exchangers to be generated before or after any existing heat exchanger in a heat recovery network. The aim is to make structural optimization more flexible and free.

Formulation of Dynamic Split Group NNM

The formulation of DSG-NNM is as follows. When there is a heat exchange unit on the first split group (of the first hot stream), a new split group is added upstream to ensure that there is a possibility for the heat exchanger newly generated to be positioned near the inlet of the stream. To achieve this, the number of split groups in the stream is incremented by one (i.e. $Nd_H + 1$ or

$Nd_C + 1$), and the serial number of a ‘hot node’ corresponding to the existing heat exchanger is adjusted accordingly. From Figure 2, the first split group on the first stream has become the second split group, and the newly generated heat exchanger has the opportunity to be placed on a node at the inlet-end region of the stream, the case which was not possible with NNM. After completing the above operations, subsequent nodes are navigated. If it is found that there are heat exchangers in two consecutive split groups, a new split group is inserted between the two split groups. As in the earlier scenario, the number of split groups is increased by one, and the node numbers are also adjusted. The node number on the hot stream is increased by Nf_H , and the node number on the cold stream is increased by Nf_C . After adding a new split group before the first split group, and inserting a new split group between the first and second, the split group which was second becomes the fourth group (Figure 2).

Figure 2: The Schematic Representation of Dynamic Split Group NNM



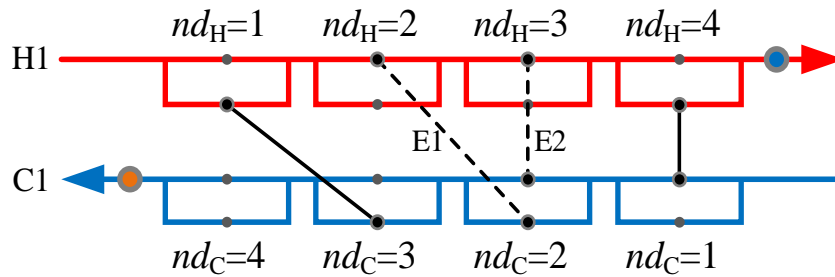
Generation of a new heat exchanger between two successive split groups becomes possible after the

following rearrangement: the split groups on the stream are moved in sequence until the last split

group is identified. If there is a heat exchanger in this last split group, a new split group is added after it. This time, the node number corresponding to the existing heat exchanger needs not be changed, the total number of split groups is incremented by one. If there is no heat exchanger at the end of the stream, it implies that the split

group rearrangement is complete for that stream and the rearrangement begins for the subsequent stream. The same operation is performed for all cold and hot streams in DSG-NNM (Figure 2). Figure 3 shows the stream matching for the two-stream case where the dotted line represents the newly generated heat exchanger.

Figure 3: A Simple Two-stream HEN with Newly Generated Exchangers (dotted lines)

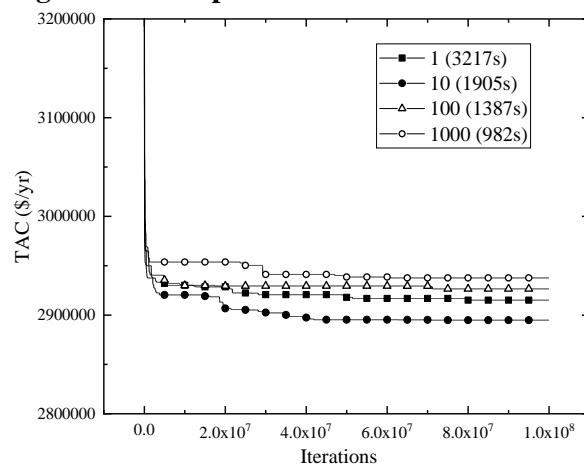


Implementation Cycle of Dynamic Split Group NNM

In the DSG-NNM, the addition of the split groups in process streams is thought to overcome the NNM's drawback of not being able to generate heat exchange units in certain regions on the streams. However, the frequent addition of split groups can result in too many split groups hence reducing computational efficiency. Therefore, it is necessary to identify the appropriate operating cycles for the addition of extra split groups.

The research reported in this paper considered operating cycles of four orders of magnitude that were executed in iterations 1, 10, 100 and 1000, and compared the changes in network costs in the respective cycles. A nine-stream problem (9SP) (comprising five cold streams and four hot streams (Pettersson, 2005)) was used as the experimental case study. The results recorded after 10^8 iterations, showed that as many execution cycles are allowed, lower (better) TACs are achieved. However, it turned out that the shorter the execution cycle, the slower the algorithm runs (Figure 4).

Figure 4: The Optimization Trend Curves for Different Operating Cycles in DSG-NNM



In this work, the operation for adding extra split groups was performed after every 100 iterations averaging the two aforementioned extremes. The

strategy was experimented with for two medium-sized examples namely 16SP (shown in Table 1) and 20SP (see Table 2).

Table 1: Stream and Cost Data for 16SP (Khorasany and Fesanghary, 2009)

Stream	T^{in} (°C)	T^{out} (°C)	C_p (kW/°C)	h (kW/m ² /°C)
H1	385	159	131.51	1.238
H2	516	43	1198.96	0.546
H3	132	82	378.52	0.771
H4	91	60	589.545	0.859
H5	217	43	186.216	1
H6	649	43	116	1
C1	30	385	119.1	1.85
C2	99	471	191.05	1.129
C3	437	521	377.91	0.815
C4	78	418.6	160.43	1
C5	217	234	1297.7	0.443
C6	256	266	2753	2.085
C7	49	149	197.39	1
C8	59	163.4	123.156	1.063
C9	163	649	95.98	1.81
C10	219	221.3	1997.5	1.377
hu1 (flue gas)	1800	800		1.2
hu2 (steam)	236	236		1
cu (water)	38	82		1

Heat exchanger cost=26600+4147.5A^{0.6}\$/yr. (A in m²)
hu1 cost=35.0 \$/kW/yr.
hu2 cost=27.0 \$/kW/yr.
cu cost=2.1 \$/kW/yr.

Table 2: Problem Data for a 20-stream Case (Wu et al., 2006)

Stream	T^{in} (°C)	T^{out} (°C)	C_p (kW/°C)	h (kW/m ² /°C)
H1	453	348	30	2
H2	553	393	15	0.6
H3	453	348	30	0.3
H4	413	318	30	2
H5	493	393	25	0.08
H6	453	328	10	0.02
H7	443	318	30	2
H8	453	323	30	1.5
H9	553	363	15	1
H10	453	333	30	2
C1	313	503	20	1.5
C2	393	533	35	2
C3	313	463	35	1.5
C4	323	463	30	2
C5	323	523	20	2
C6	313	423	10	0.06
C7	313	423	20	0.4
C8	393	483	35	1.5
C9	313	403	35	1
C10	333	393	30	0.7
HU (stream)	598	598	-	1
CU (water)	298	313	-	2

Cost data:

Annual cost of heat exchangers = 8000 + 800 A^{0.8} \$.a⁻¹ (A in m²)Annual cost of hot utility (stream) = 70 \$.kW⁻¹.a⁻¹Annual cost of cold utility (water) = 10 \$.kW⁻¹.a⁻¹

HEN Optimization Using RWCE

The optimization of HENs based on DSG-NNM using RWCE (Xiao and Cui, 2017) involved a

pool of M networks (i.e. HENs). Each HEN evolved by virtue of the heat duties of individual heat exchangers placed on defined nodes according to Equation (31)

$$\vec{Q}'_{m,n} = \begin{cases} \vec{Q}_{m,n} & \text{if } rand > wp \\ \vec{Q}_{m,n} + (1 - 2\alpha_1) \cdot \Delta Q \cdot \mu_1 & \text{if } rand \leq wp \end{cases} \quad (31)$$

Where wp is the random walk probability, α_1 and μ_1 are random numbers in the interval (0,1) and ΔQ is the maximum change of heat load in the direction determined by $(1 - 2\alpha_1)$. $m = 1, 2, \dots, M$ and $n = 1, 2, \dots, N$ are respectively, the indices for networks in RWCE population and heat exchangers in an individual network.

Since stream splits were allowed in the study, the evolution of HENs also included the updates of split fractions that accounted for the proportions of branches of streams. Split fractions evolved according to Equations (32) and (33),

$$\overrightarrow{sfH}'_{m,n} = \begin{cases} \overrightarrow{sfH}_{m,n} & \text{if } rand > \delta f \\ \overrightarrow{sfH}_{m,n} + (1 - 2\alpha_2) \cdot \Delta L \cdot \mu_2 & \text{if } rand \leq \delta f \end{cases} \quad (32)$$

$$\overrightarrow{sfC}'_{m,n} = \begin{cases} \overrightarrow{sfC}_{m,n} & \text{if } rand > \delta f \\ \overrightarrow{sfC}_{m,n} + (1 - 2\alpha_3) \cdot \Delta L \cdot \mu_3 & \text{if } rand \leq \delta f \end{cases} \quad (33)$$

where, ΔL is the maximum possible change of split fractions, and α and μ are random numbers in the interval (0,1). δf is the threshold value for allowing split fractions to change during the optimization process.

A new heat exchanger was generated if the corresponding heat load/duty Q was such that $Q > Q_{\min}$ and an existing heat exchanger would be eliminated if its heat duty Q goes as low as $Q < Q_{\min}$. In that line, RWCE achieved the simultaneous optimization of continuous variables and integer variables. All HENs were evaluated in terms of their TACs (according to Equation (5)) after the inclusion of all necessary utility requirements for each network. Note that RWCE permits less economical HENs at some minute probability δ_{up} (of orders of 10^{-2}) in order to avoid being trapped in local optimality. After many fitness evaluations, the HEN that exhibits the lowest TAC, upon reach of the set termination criterion, is considered the best solution.

RESULTS AND DISCUSSION

Case 1 (16SP)

Case 1 involves 16SP proposed by Khorasany and Fesanghary (Khorasany and Fesanghary, 2009). This case contains ten cold streams and six hot streams (Table 1). In this study, the optimization of HENs using RWCE based on DSG-NNM yielded a final network structure shown in Figure 5, with a TAC of \$6,859,471, which surpasses many literature solutions (Table 3). As seen in the figure, there are seven process exchangers on hot stream H2. Essentially, as the heat content on hot stream H2 is very large and hot streams are lesser in number than cold streams, splitting of the stream H2 is inevitable and multiple splits can exist. The splitting of hot stream H2 (in this work) allowed a greater exchange of heat thus, minimizing utility loads. This solution (Figure 5) requires cold water to accomplish the necessary cooling of hot streams H2, H3, H4, H5 and H6 and needs the use of flue gas for heating of cold streams C3 and C9 to target temperatures.

Khorasany and Fesanghary (2009) used a harmony search algorithm and obtained a HEN costing \$1,420,672/yr less than the cost exhibited by the existing plant. Huo and coworkers (Huo et al., 2013) optimized SWS-based HENs for 16SP using a hybrid method of GA and PSO and reported a solution with stream splits whose cost

is displayed in Table 3, along with costs of other literature solutions. Although the solution has a slightly higher utility requirement compared to those presented by Bao et al. (2018) and Pavão et al. (2018), it exhibits one unit less compared to theirs (Table 3), implying savings in installation and operating costs.

Table 3: Solutions for Case 1 (16SP)

Reference	Units	$Q_{hu}(kW)$	$Q_{cu}(kW)$	TAC(\$/yr)
Khorasany and Fesanghary, (2009)	18	66,070	469,620	7,435,740*
Huo et al., (2013)	16	38,800	442,370	7,361,190*
Zhang et al., (2017)	19	23,790	427,360	7,212,115 ^{rc}
Pavão et al., (2018)	19	10,470	414,030	6,801,261*
Bao et al., (2018)	19	10,050	413,610	6,869,610*
This work (Figure 5)	18	14,499	418,062	6,859,471*

^{rc}Revised cost(Pavão et al., 2017a)

*Solution with stream splits incorporated

Case 2 (20SP)

This case study is a twenty-stream problem (20SP), first proposed by Wu et al. (2006), which includes ten hot streams and ten cold streams as detailed in Table 2. The authors used the traditional temperature-enthalpy diagram method to solve this case and obtained a HEN with a TAC of 1,827,772 \$/yr. Luo et al., (2009) applied a hybrid algorithm which effectively avoids the premature convergence of the genetic algorithm by adding the SA algorithm, and obtained a lower

TAC of 1,753,271 \$/yr. A flexible process-stream grouping method was adopted by Laukkanen et al. (2012) for HEN design and a network structure with a TAC of 1,739,778 \$/yr was reported. Pavão et al. (2017) modelled the HEN synthesis problem using stage-wise superstructure and used a heuristic algorithm to solve 20SP. This algorithm combines SA and RFO algorithms to avoid being trapped in local optimal solutions. The network structure they obtained has a TAC of 1,725,725 \$/yr.

Figure 5: Solution for 16SP with a TAC of 6,859,471 \$/yr Obtained Using RWCE Based on DSG-NM. Heat Duties are in kW and Stream Split Fractions are Enclosed in Parentheses.

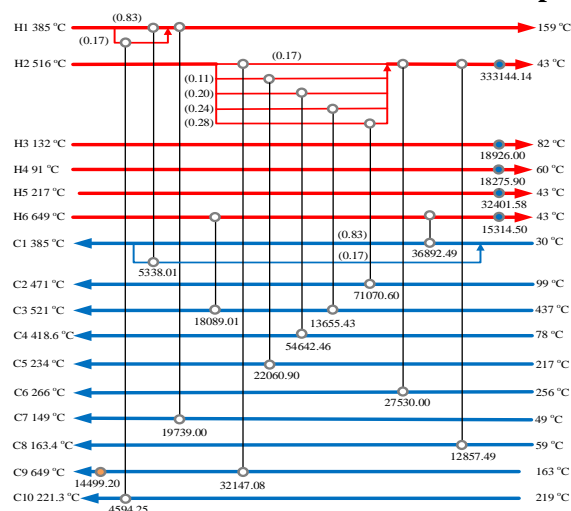


Figure 6: Solution for 20SP with a TAC of 1,725,074 \$/yr Obtained Using RWCE Based on DSG-NNM. Numbers Against Each Match are Exchanger Duties, in kW and Split Fractions are in Parentheses.

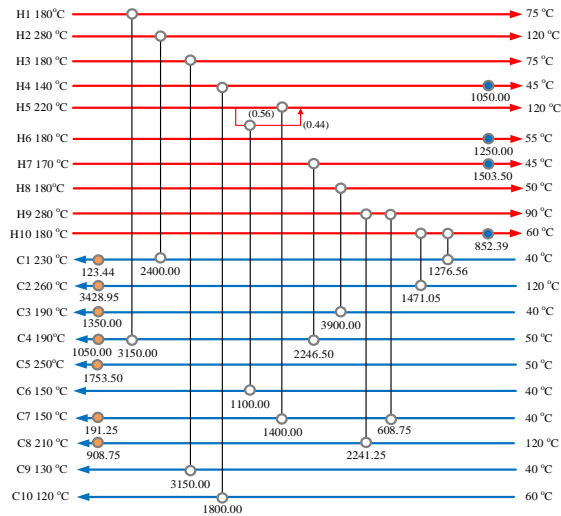


Table 4: Solutions for Case 2 (20SP)

Reference	Units	Q_{hu} (kW)	Q_{cu} (kW)	TAC(\$/yr)
Luo et al., (2009)	26	9,013	5,363	1,753,271
Laukkanen et al., (2012)	24	9,500	5,350	1,739,778
Pavão et al., (2017b)	24	-	-	1,725,295
This work (Figure 6)	25	8,805	4,655	1,725,074

In this work, HEN optimization was done using RWCE and DSG-NNM achieving a final network that costs 1,725,074 \$/yr, and its structure is shown in Figure 6. The comparison of this solution with the networks published in the literature is shown in Table 4. Since the number of nodes can flexibly be increased, DSG-NNM influences competition among heat exchange units. Furthermore, incorporating additional split groups in the NNM broadens the solution space for the RWCE, providing an effective global optimization mechanism. Utilizing the algorithm for HEN optimization based on the modified model (DSG-NNM) facilitates the structural evolution of HENs, resulting in enhanced global optimal structures.

The HEN structure in Figure 6, obtained with the use of DSG-NNM, has a lower TAC (by 221 \$/yr) compared with the structure of the literature's best solution (Pavão, et al., 2017b). Also, the network exhibits only one split while the network structure presented by Pavão et al. (2017b) has six splits, which considerably increases the complexity of the HEN structure. The decrease in the number of

splits is accounted for by the newly added nodes, which expand the solution space. The splits in the original structure are more likely to be replaced by newly generated heat exchange units in the added nodes, thus leading to better HEN configurations.

CONCLUSION

In this paper, an improved version of NNM (i.e. a dynamic split group NNM (DSG-NNM)) has been proposed, which makes it possible for new heat exchange units to be generated before or after any existing heat exchangers to enhance structural optimization and network flexibility. Random-walk algorithm with compulsive evolution (RWCE) was used to optimize the heat exchanger networks (HENs). Two case studies were solved to test the effectiveness of DSG-NNM, and the results indicate that the proposed operation is attractive for more appropriate exchanger connections and can improve the topological evolutions of HENs and therefore, produce HENs that are more economical. The costs of the final networks obtained in the study are superior to many previously published solutions. Although the DSG-NNM plays a positive role in enhancing

the optimization quality of HENs, the frequent addition of split groups in process streams reduces computational efficiency.

Future work could involve case studies with a greater number of process streams to further evaluate the model's applicability for large-scale HEN synthesis problems. Additionally, exploring

hybrid algorithms that leverage the RWCE alongside other optimization techniques could enhance the optimization of HENs based on the framework of DSG-NNM.

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Nomenclature

<u>Abbreviations</u>	$\mu, \alpha, rand$ random numbers in the interval (0,1)
HEN heat exchanger network	z binary variable
TAC total annual cost	<u>Parameters</u>
SWS stage-wise superstructure	CpH, CpC heat capacity flow rates of hot/cold streams, kW/°C
NNM node-based non-structural model	hH, hC heat transfer coefficients of hot/cold streams, kW·m ⁻² ·°C ⁻¹
RWCE random walk algorithm with compulsive evolution	h_{hu}, h_{cu} heat transfer coefficients of hot/cold utilities, kW·m ⁻² ·°C ⁻¹
H, C hot, cold process streams	U overall heat transfer coefficient for heat units, kW·m ⁻² ·°C ⁻¹
<u>Superscripts</u>	T_H^{in}, T_C^{in} supply temperatures of hot/cold streams, °C
in inlet	T_H^{out}, T_C^{out} target temperatures of hot/cold streams, °C
out outlet	Thu, Tcu temperatures of hot/cold utilities, °C
<u>Subscripts</u>	CFe fixed charge of heat exchanger, \$/yr
i index for hot process streams	CAe area cost coefficient of heat exchanger, \$/yr
i' index for cold process streams	CFh, CFc fixed charge of hot/cold utilities, \$/yr
hu hot utility	CAh, CAc area cost coefficients of heaters/coolers, \$/yr
cu cold utility	CHU, CCU cost coefficients of hot/cold utilities, \$/yr
nd_H, nd_C index for split groups in hot, cold streams	β exponent for area
nf_H, nf_C index for substreams in hot, cold streams (in NNM)	Q_{min} minimum heat load of an exchanger, kW
nd_H, nd_C index for split groups in hot, cold streams	δ_{up} probability of accepting non-improving solutions
<u>Variables</u>	ΔQ maximum change of heat load, kW
fH, fC split fractions of hot/cold substreams	<u>Sets</u>
Q heat load of process exchanger, kW	N_H, N_C number of hot, cold streams
Q_{cu}, Q_{hu} heat duty of cooler/heater, kW	$Ne_i, Ne_{i'}$ number of nodes on hot stream i , cold stream i'
Aep area of a process heat exchanger, m ²	Nd_H, Nd_C number of split groups on hot, cold streams
Aec, Aeh area of outlet-end cooler/heater, m ²	Nf_H, Nf_C number of split branches on hot, cold streams
Aih, Aic area of inner heater/cooler, m ²	Nt_H, Nt_C total number of nodes on hot, cold streams
ΔT_{lm} logarithmic mean temperature difference, °C	<u>Matrices</u>
T temperature, °C	$\vec{Q}_{m,n}, \vec{Q}'_{m,n}$ heat loads of the m^{th} individual before, after random walk, kW
DTL, DTR temperature difference on hotter, colder side of an exchanger, °C	$\vec{sfH}_{m,n}$ split fractions for hot streams
TH, TC temperatures of hot/cold substreams, °C	$\vec{sfC}_{m,n}$ split fraction for cold stream

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